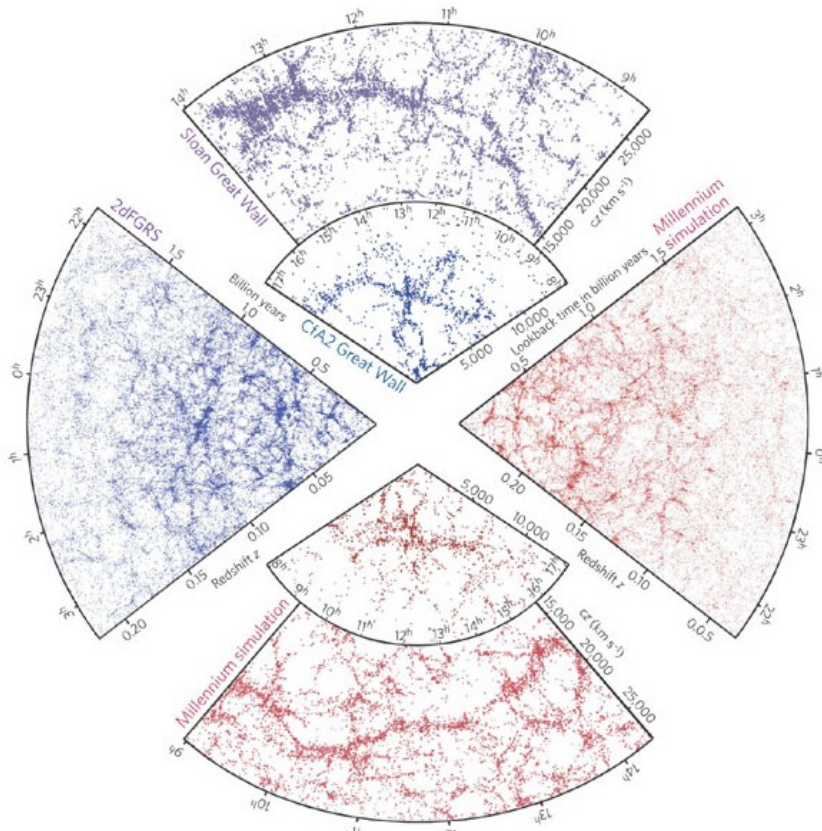


# New Trends in Computational Cosmology (Structure Formation)

**Mark Vogelsberger (Harvard/CfA)**

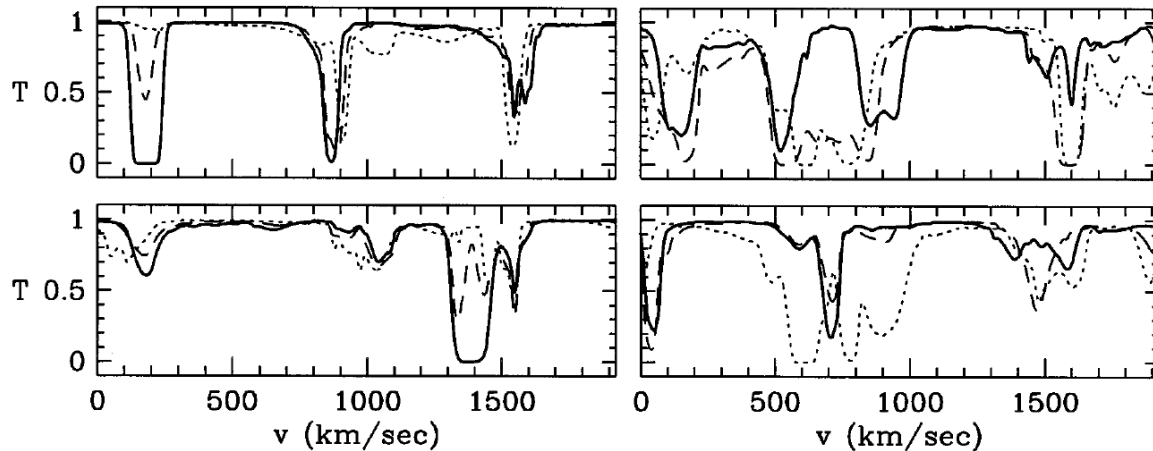
Lars Hernquist, Dusan Keres, Debora Sijacki, Volker Springel, Simon White

# Simulations of Structure Formation



## N-body large scale structure

## Springel et al (2005)



## hydrodynamics

### Ly $\alpha$ forest

Hernquist et al (1996)

# Outline

(1) N-body geodesic deviation equation:

*dark matter on small scales*

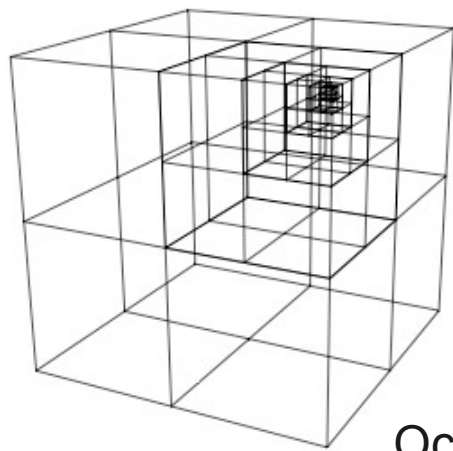
(2) Quasi-Lagrangian finite volume hydrodynamics:

*galaxy formation on a moving mesh*

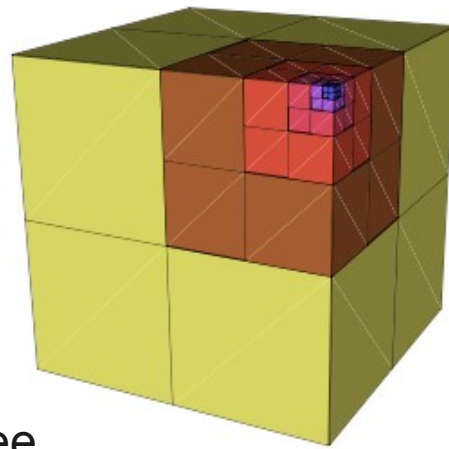
**N-body geodesic deviation equation**

***dark matter on small scales***

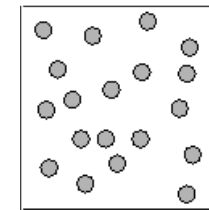
# N-body: Numerical Methods



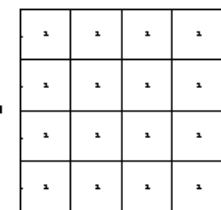
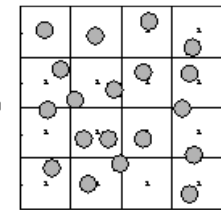
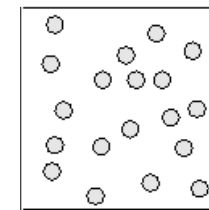
Octree



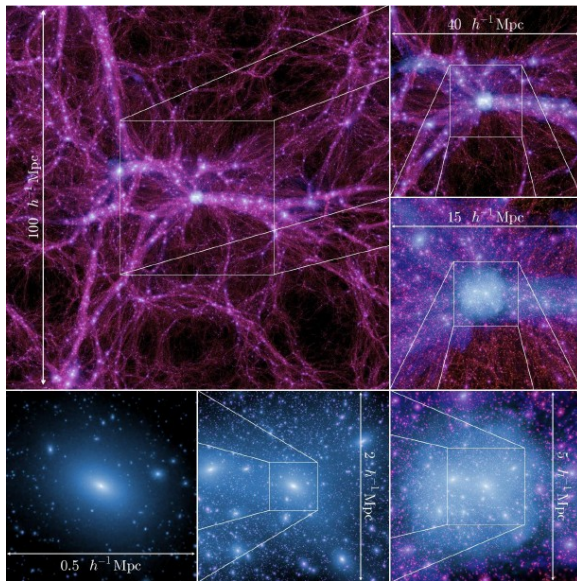
+



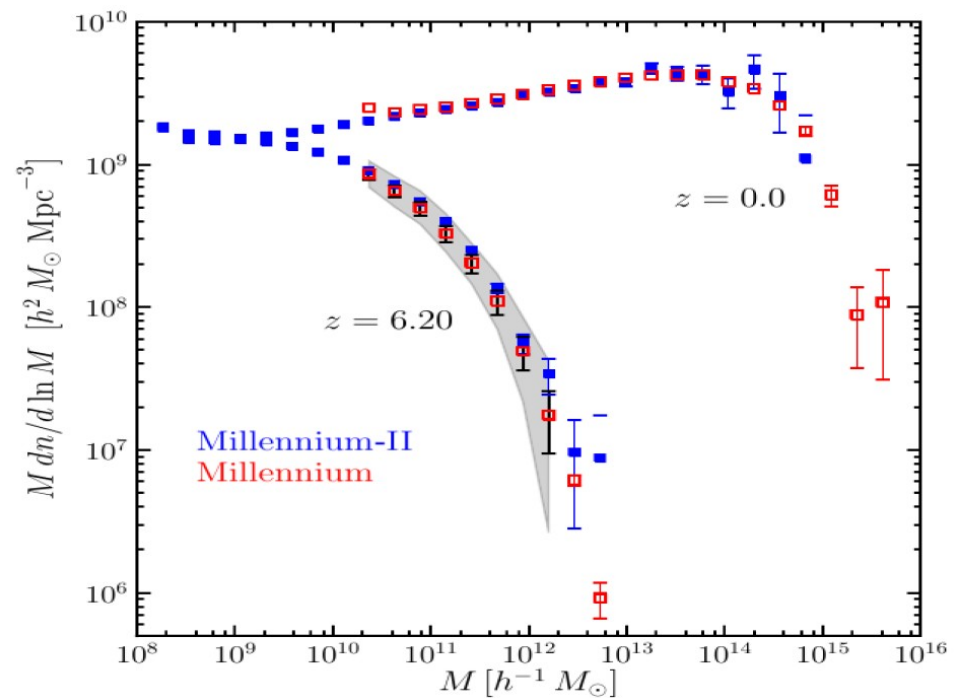
PM/FFT



= TreePM scheme [short- / long-range force split]



Boylan-Kolchin et al (2009)





# N-body: State-of-the-Art

Angulo et al (submitted)

**303 billion particles**

$L = 3 \text{ Gpc}/h$

~700 million halos  
at  $z=0$

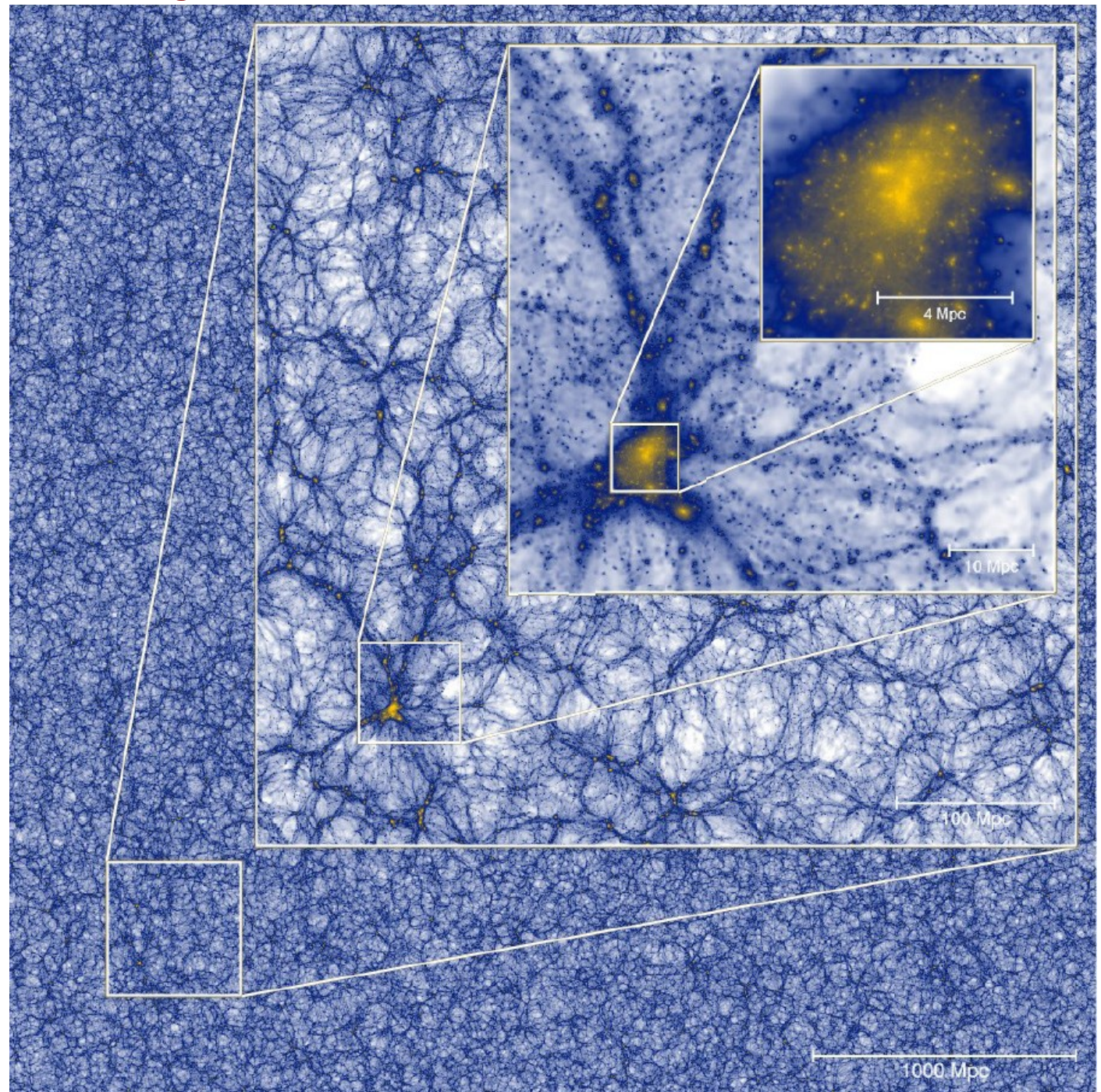
~25 billion (sub)halos  
in mergers trees

$m_p = 6.1 \times 10^9 M_\odot/h$

12288 cores,  
30 TB RAM on  
Supercomputer  
JuRoPa in Juelich

2.7 million CPU-hours

[Springel]

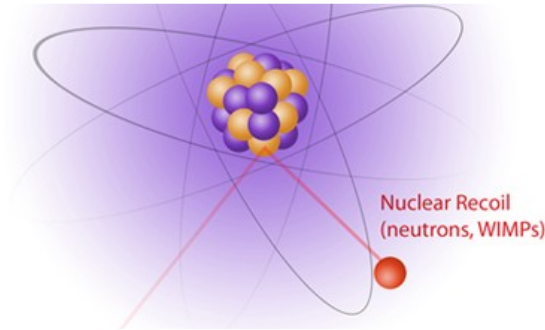




# The Hunt for Dark Matter

## Direct searches: nuclear recoil events

CRESST, XENON, ZEPLIN,  
EDELWEISS, CDMS, DAMA, ...



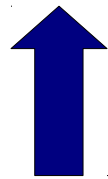
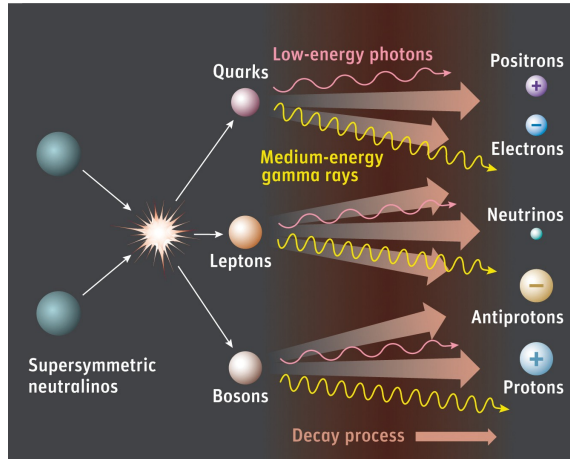
## Accelerator searches: producing DM

LHC



## Indirect searches: annihilation products

FERMI, PAMELA, ...



## Usually assumed astrophysical input:

Density:  $\sim 0.3 \text{ GeV} / \text{c}^2 / \text{cm}^3$

Velocity: Maxwellian

## Standard Halo Model (SHM):

- Smooth mass distribution
- Smooth velocity distribution
- 'Featureless' phase-space

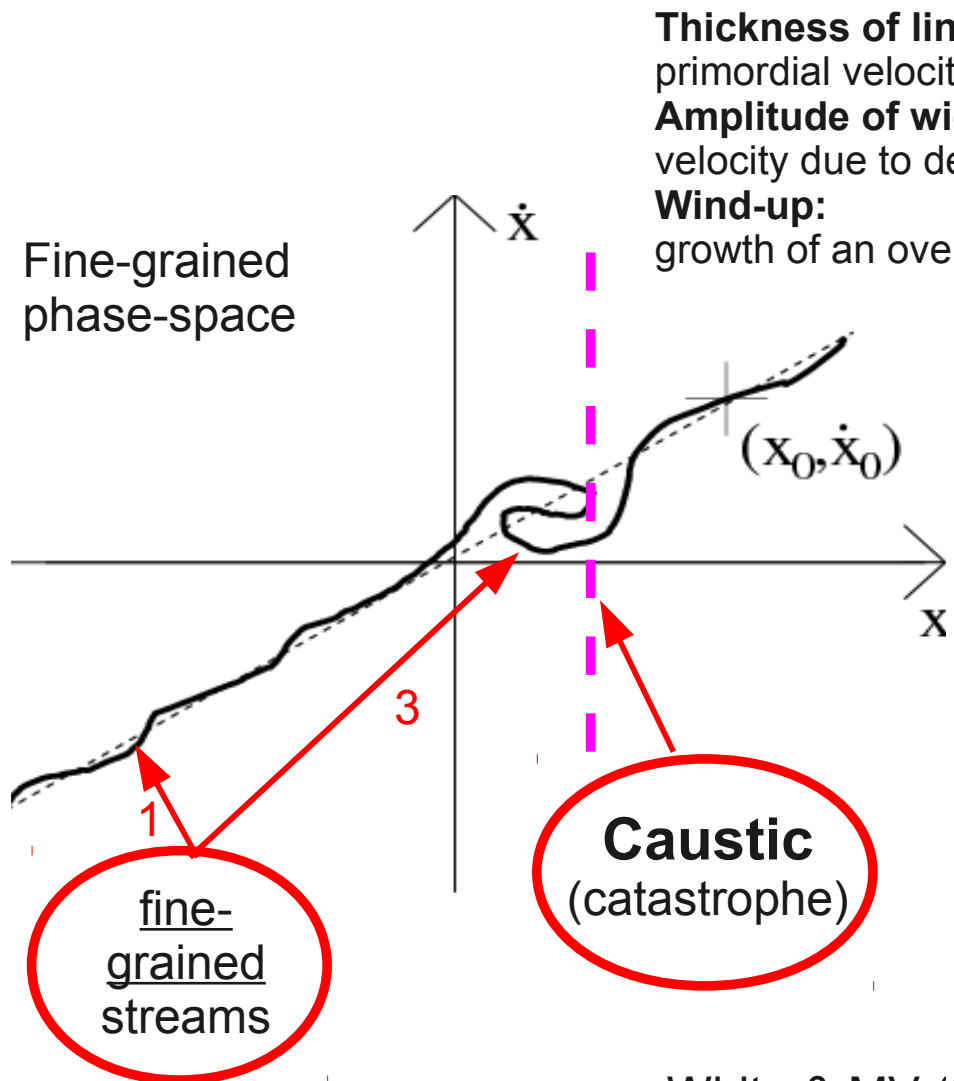


# CDM – Small Scales

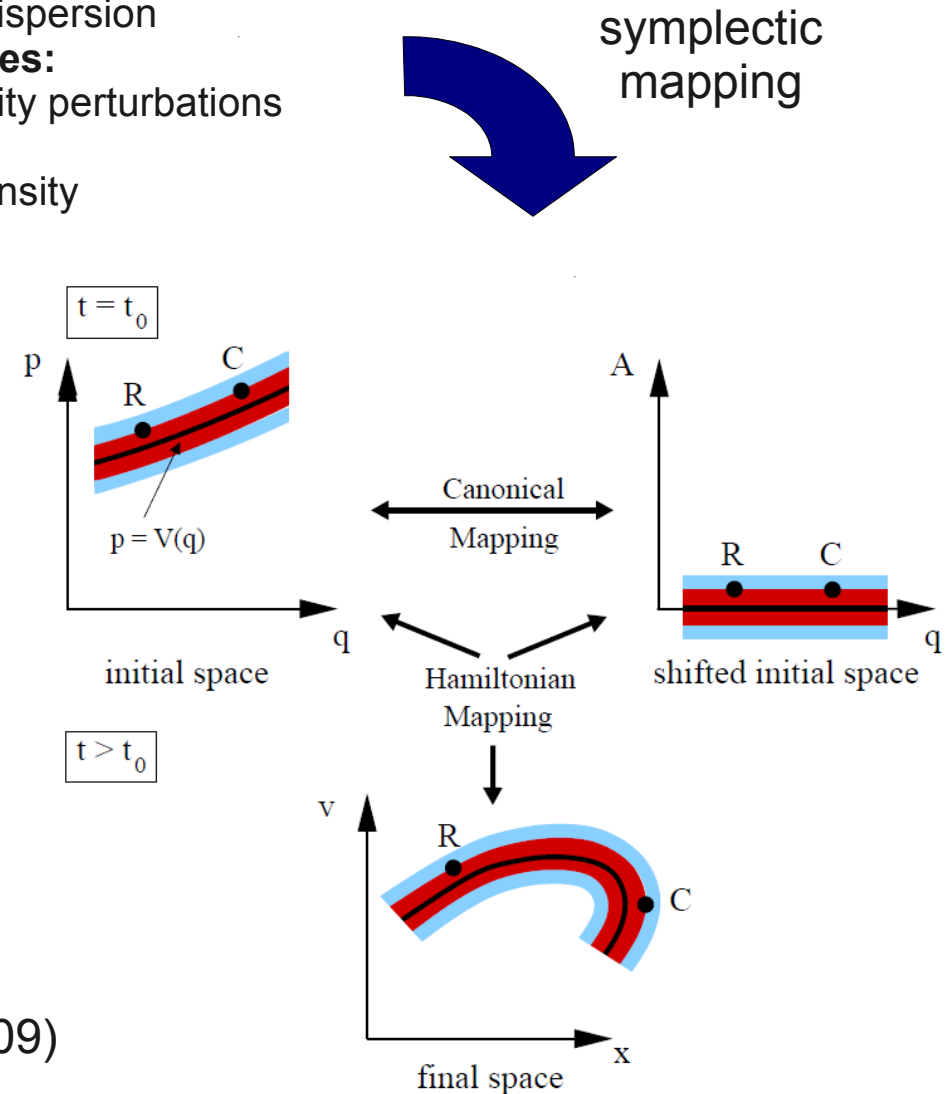
CDM is **cold** and **collisionless**



**CDM restricted to 3D hypersurface in 6D phase-space**



White & MV (2009)





# Analytic Predictions

## Self-similar halo formation:

Fillmore & Goldreich (1984), Bertschinger (1985),  
Mohayaee & Salati (2008);  
Mohayaee et al (2006); ...

## Caustic ring model:

Duffy & Sikivie (2008);  
Natarajan & Sikivie (2008);  
Onemli & Sikivie (2007);  
Natarajan & Sikivie (2007);  
Sikivie et al (1997); ...

## General arguments:

Hogan (2001)

### Predictions

- ~100 streams at solar position
- significant annihilation boost
- strong caustic rings
- discrete velocity distribution
- distinct caustic structures

→ significant effects  
on search experiments

## How realistic are these models?

Caustic densities? Number of streams? Boost factor?

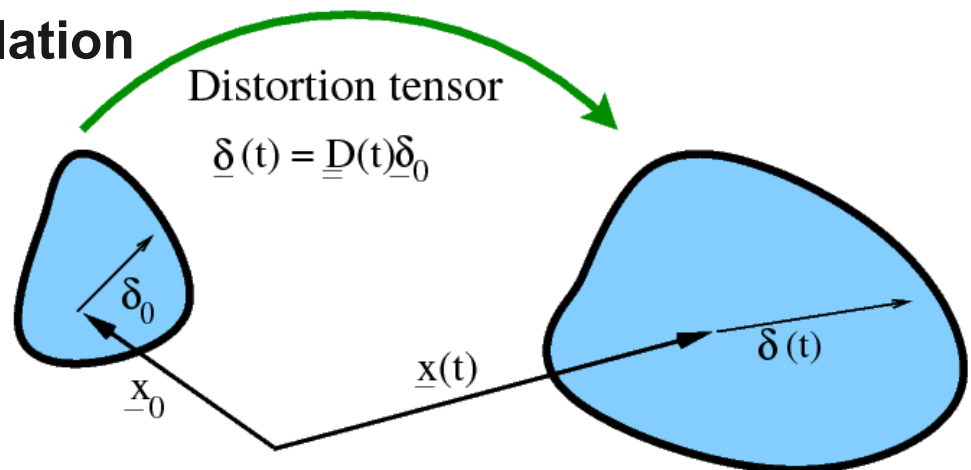
# Resolving Fine-Grained Structure with N-body Simulations

Problem: N-body simulations have too coarse phase-space sampling  
(→ missing many orders of magnitude in mass resolution/particle number)

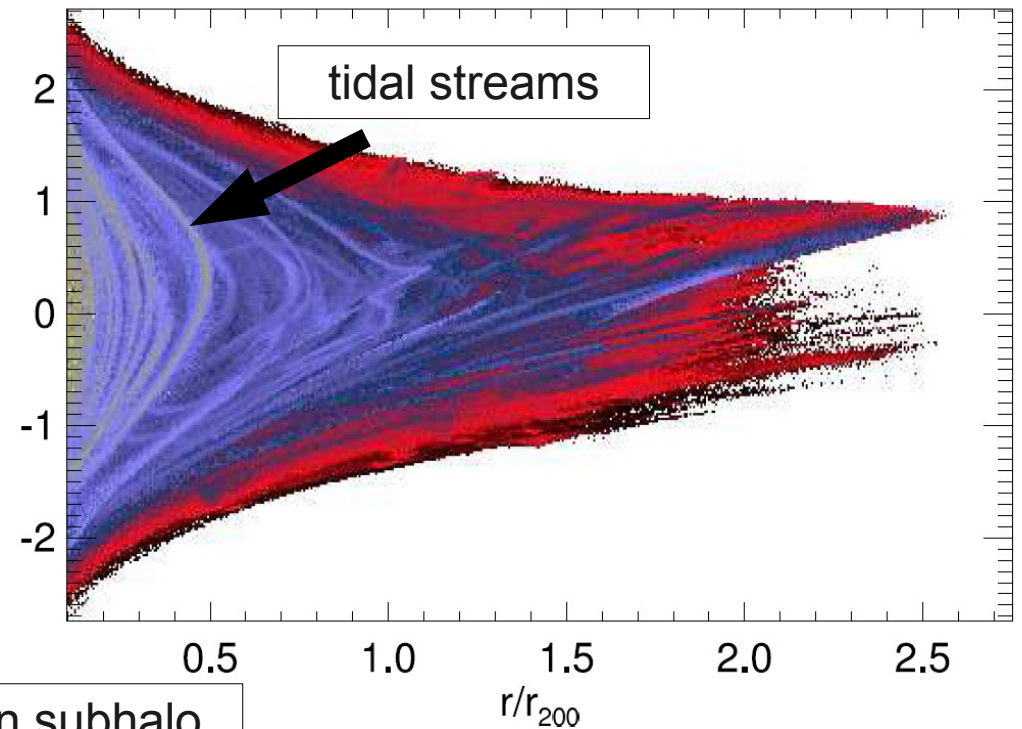
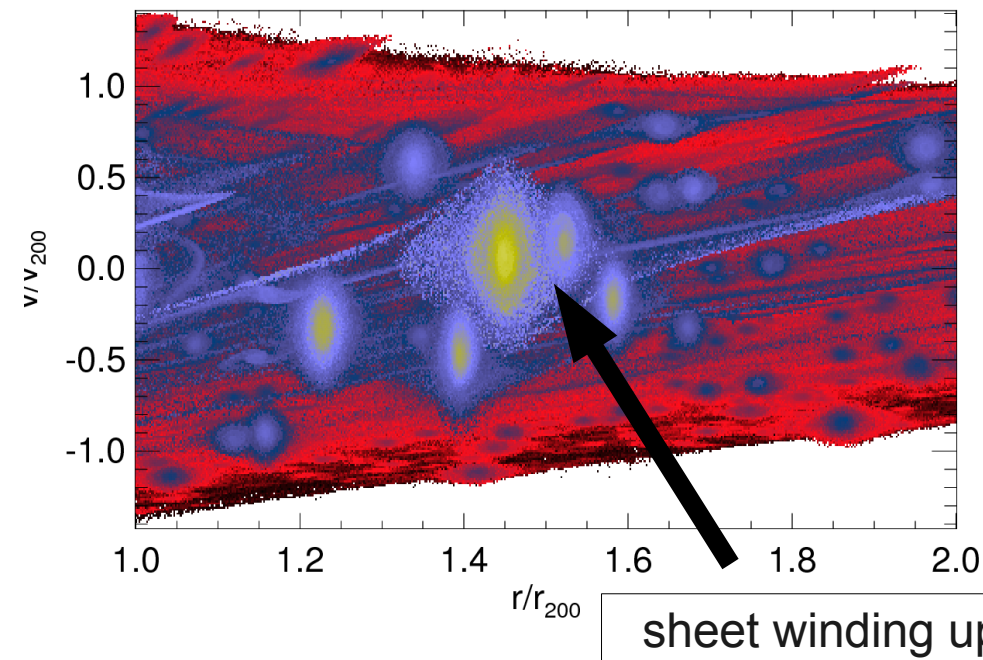
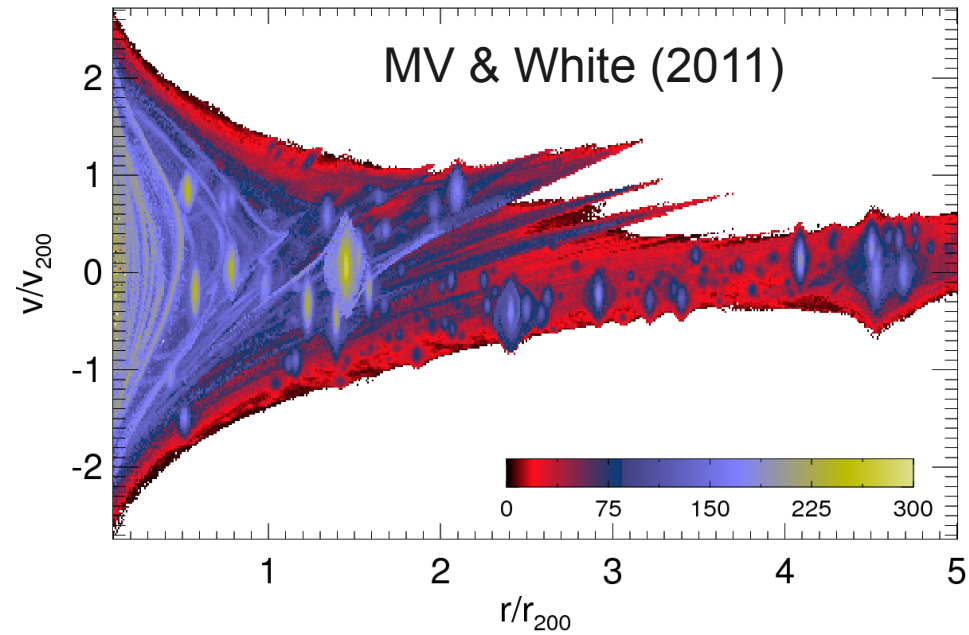
Solution: Follow the local phase-space evolution for each particle  
(→ with a phase-space geodesic deviation equation)

- calculation of **stream density**
  - **identification of caustics**
  - Monte-Carlo estimate for **intra-stream annihilation**
- allows **caustic annihilation** calculation

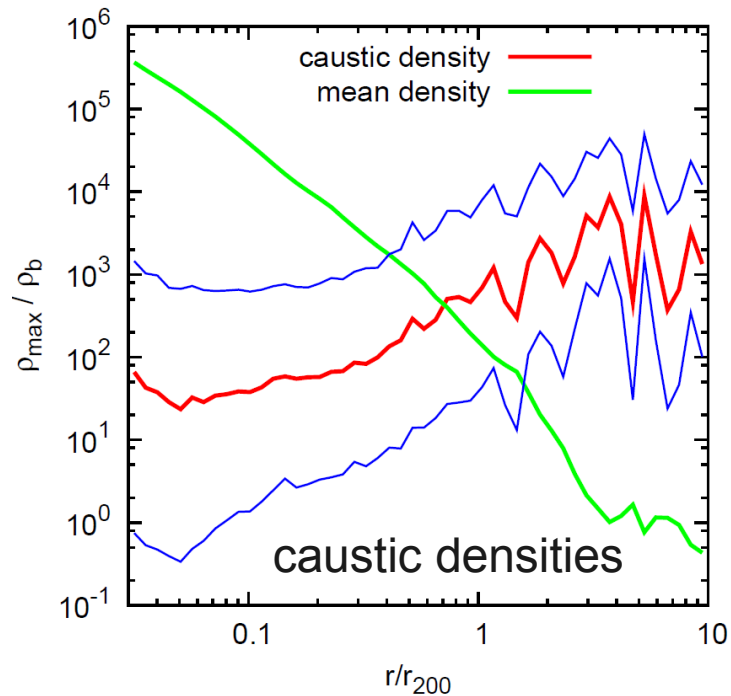
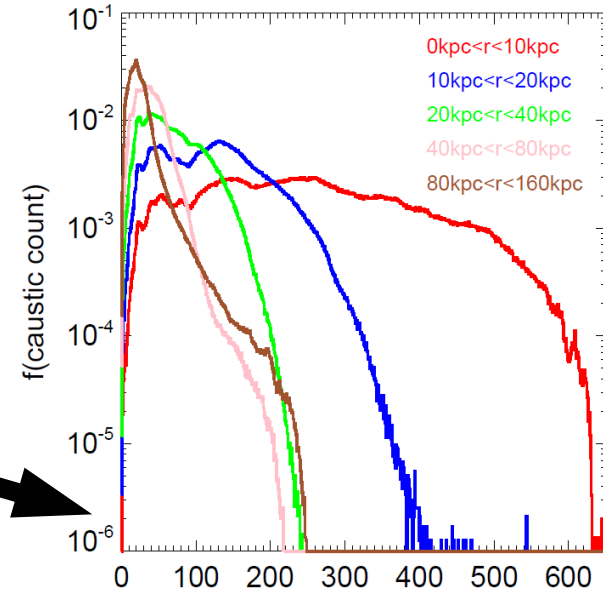
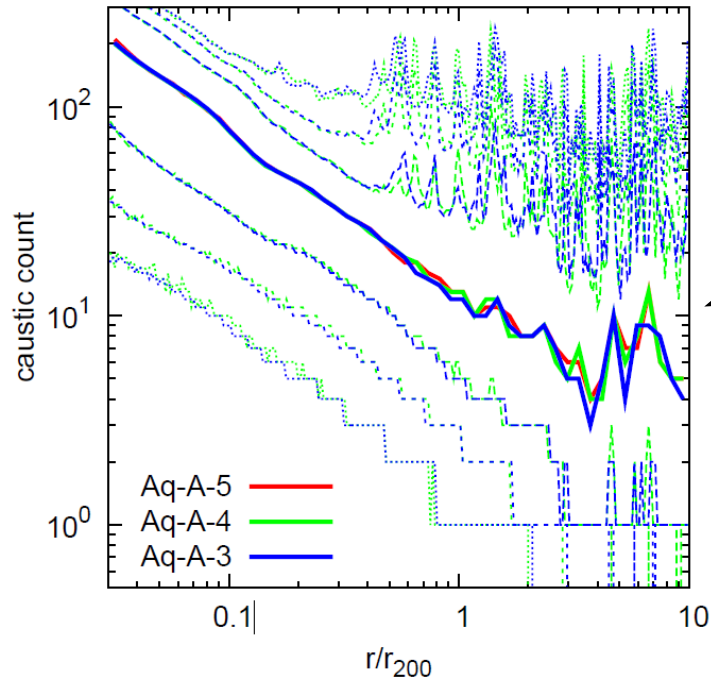
MV et al (2008)



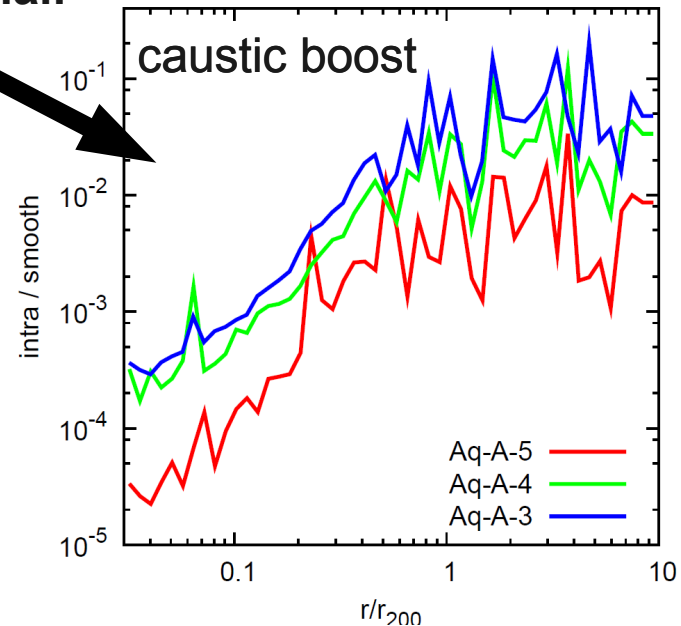
# Fine-grained Structure of Milky Way like Halos



# Fine-grained Caustics

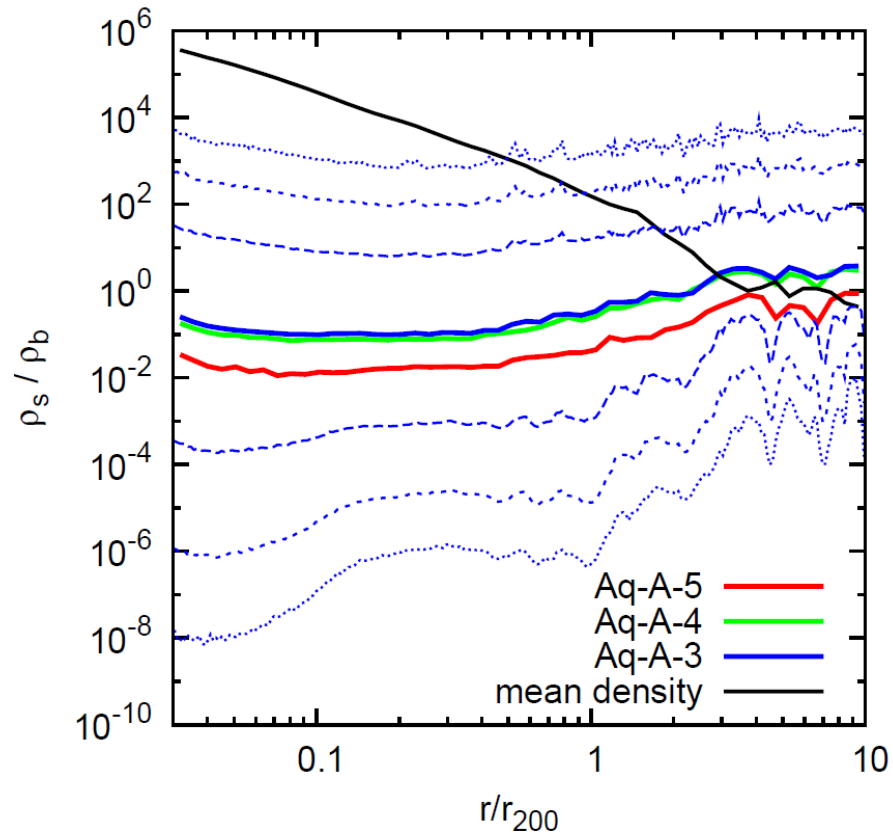


caustic **boost**  
**very small**

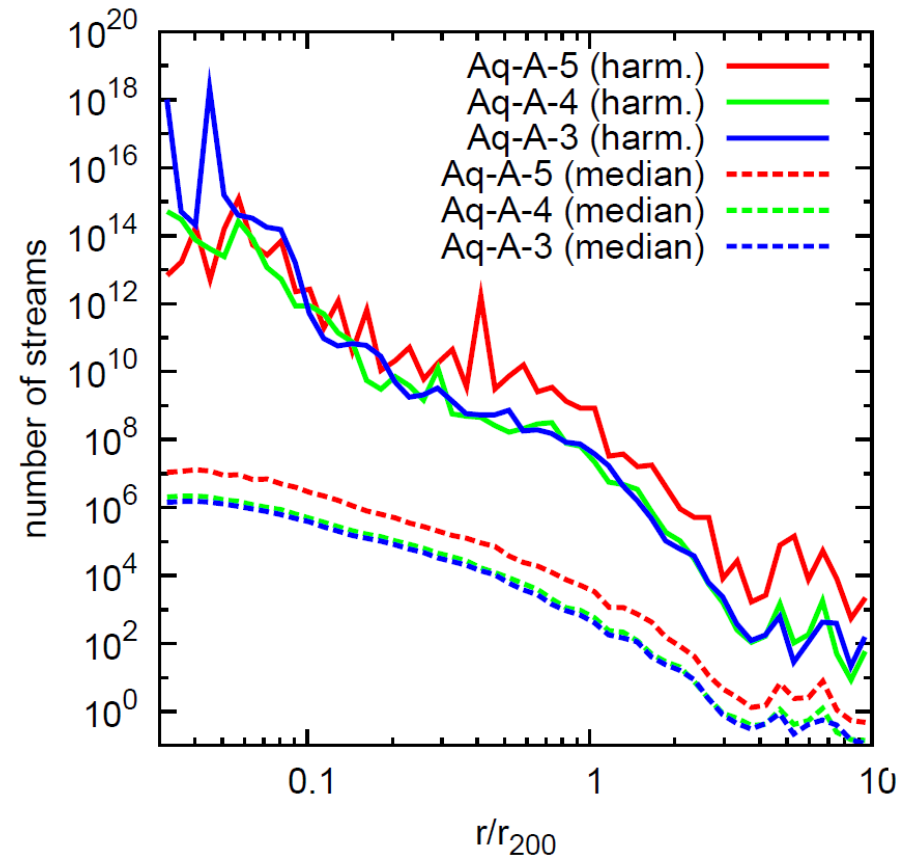




# Fine-grained streams

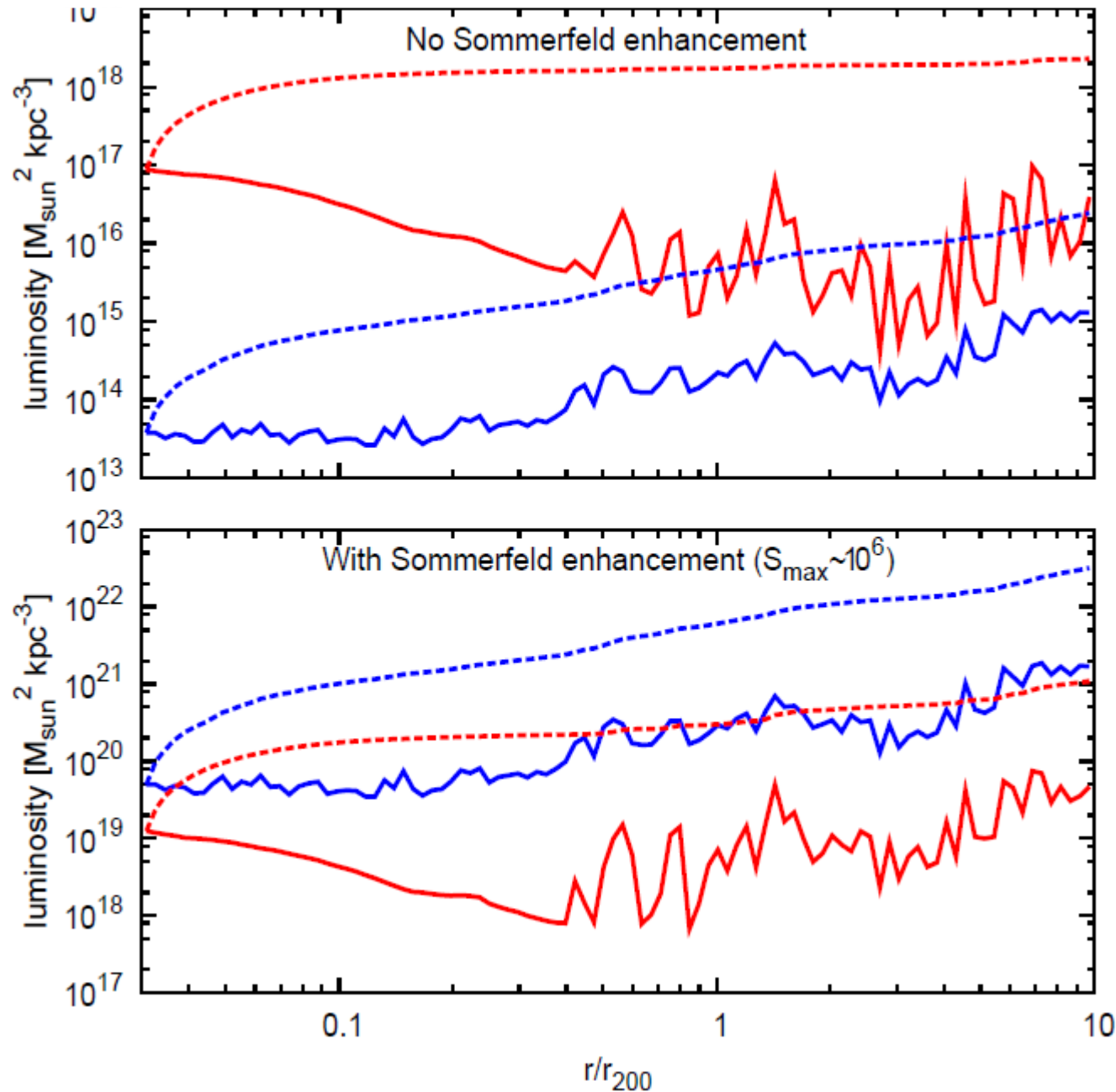


stream densities **converged**



huge number of streams in inner halo

# Sommerfeld Enhancement



**regions of low velocity  
dispersion dominate  
annihilation signal:**

→ streams contribute a lot

→ caustics negligible in  
such a scenario

red: smooth halo  
blue: streams

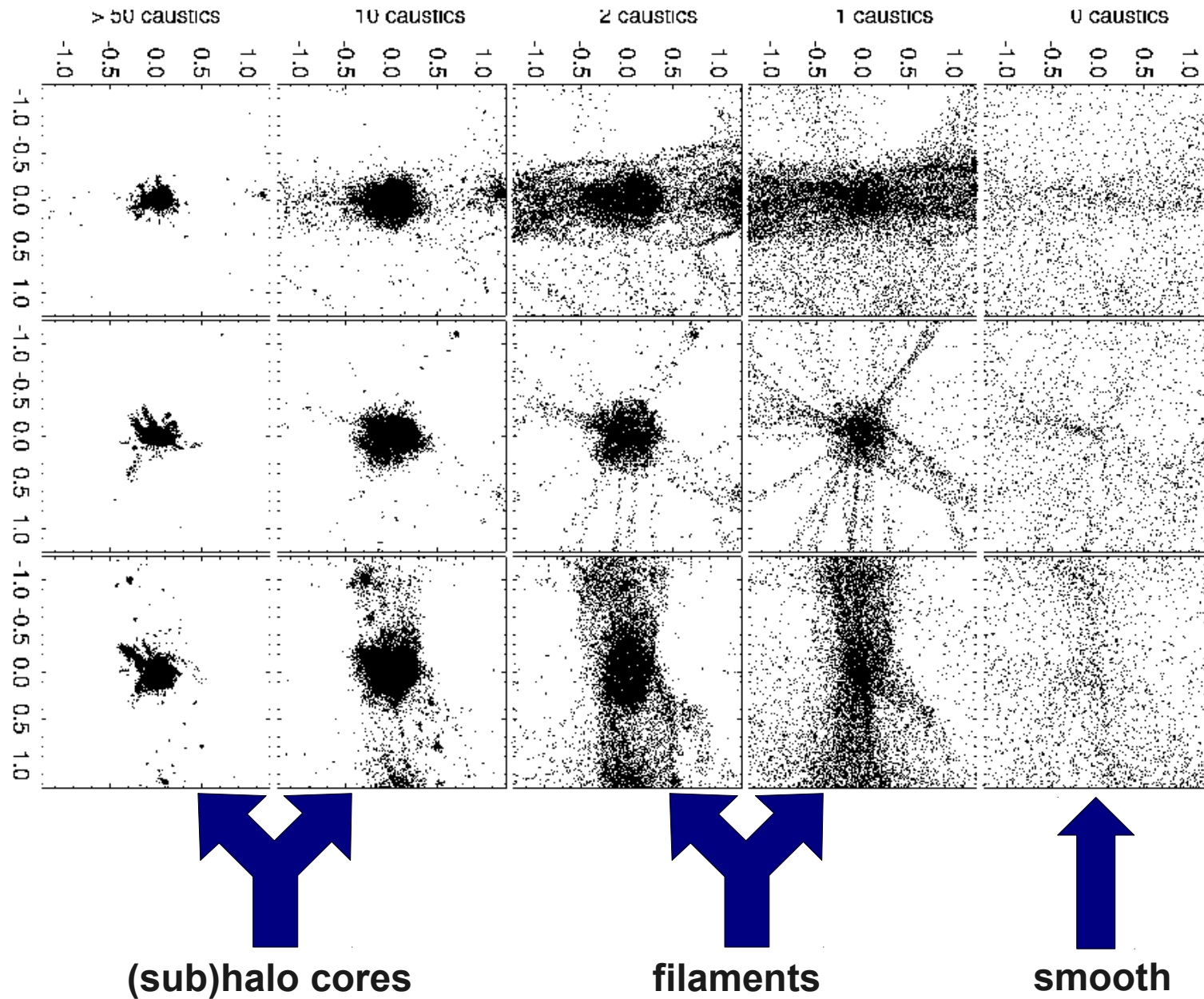
# Implications

- no massive streams near the Sun
- no discrete velocity distribution
- $\gg 100$  streams near the Sun
- no dense caustic structures
- N-body simulations do not miss much caustic annihilation



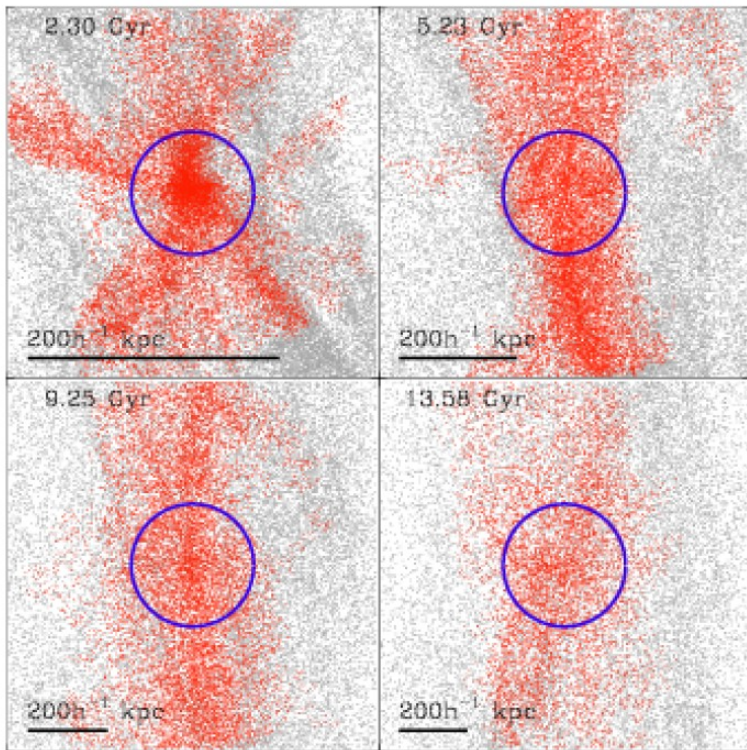
**Local DM phase-space distribution is very smooth!**

# Further Applications: Filtering the Cosmic Web

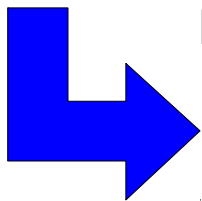
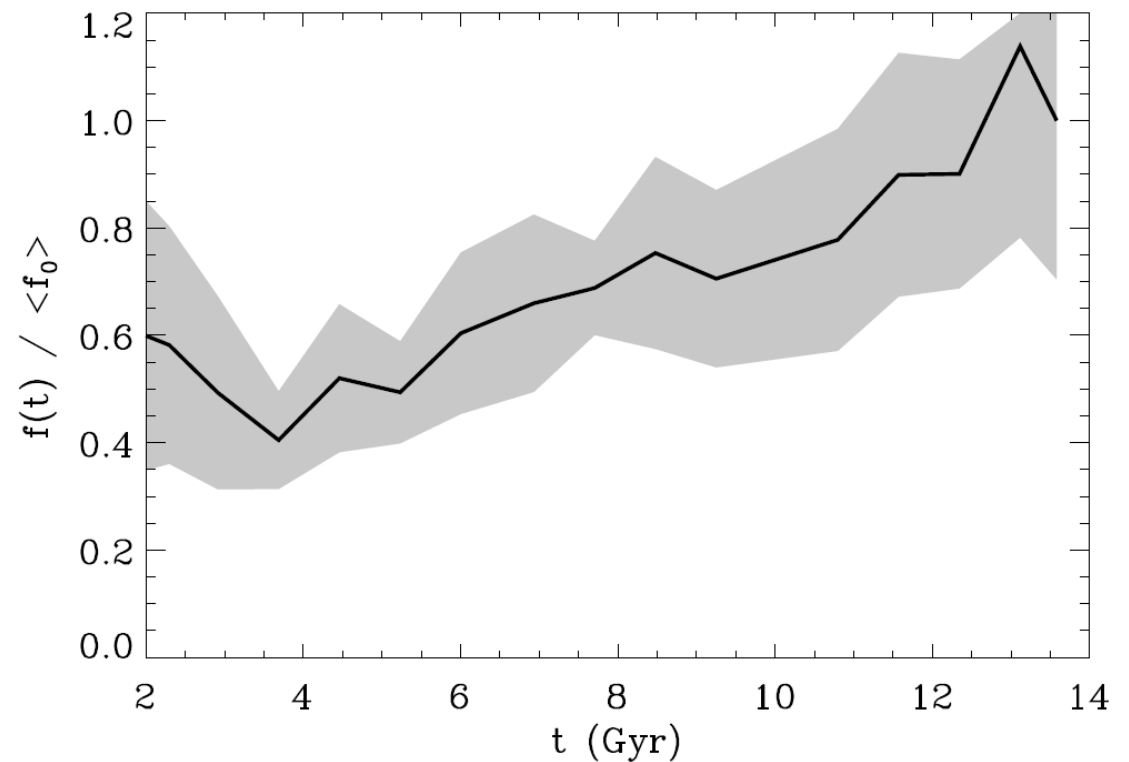




# Filaments around Halos



filament size relative to virial  
radius increase with time  
→ later infall from filaments with  
larger cross-section



FoF on low caustic  
count particles

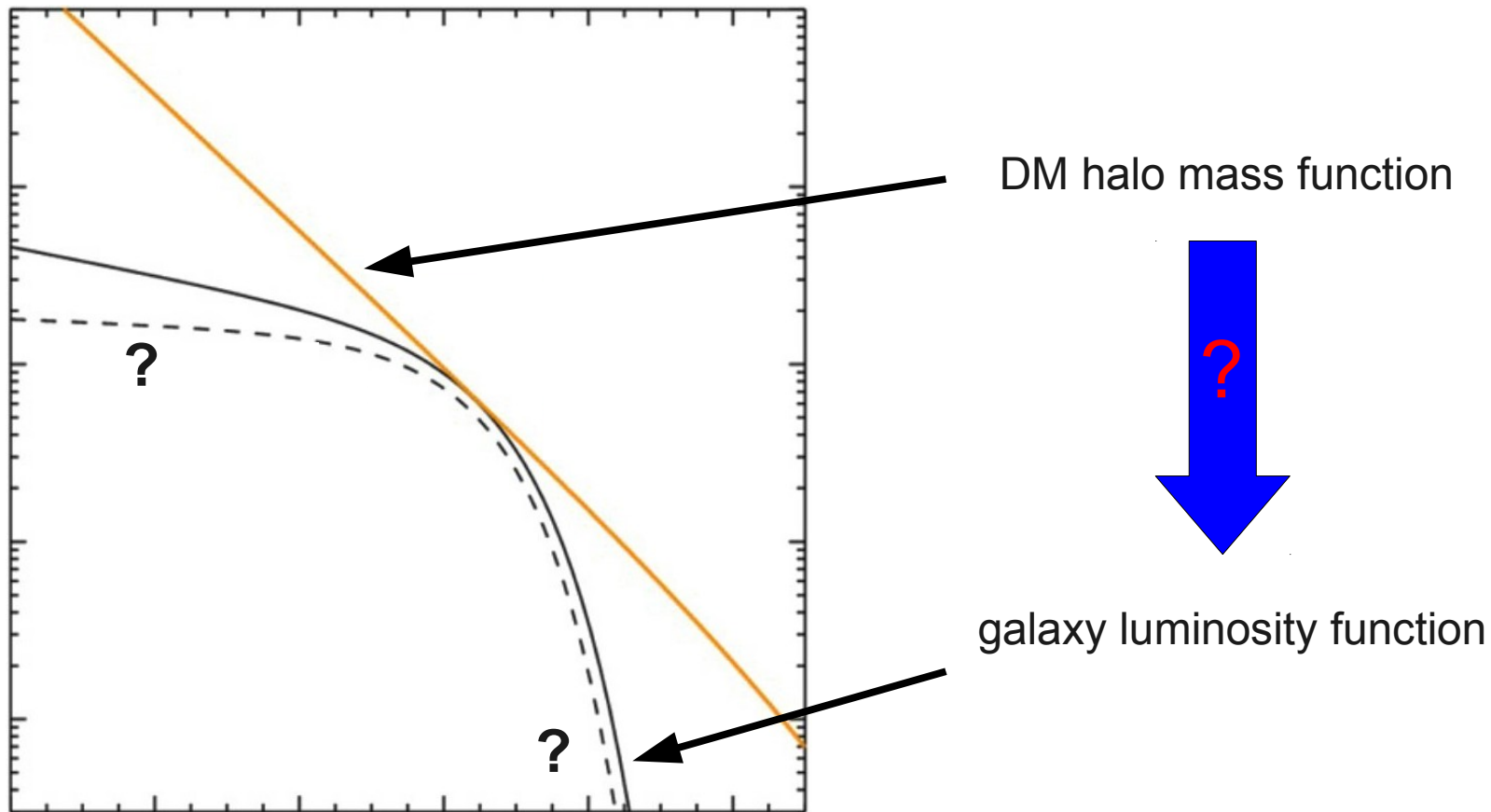
Vera-Ciro et al (2011)

# Quasi-Lagrangian finite volume hydrodynamics

*galaxy formation on a moving mesh*

# Galaxy Formation Problem: Dark Matter $\leftrightarrow$ Baryons

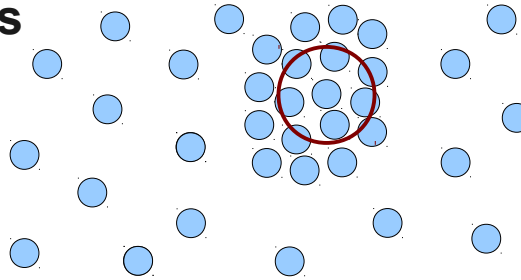
modeling baryon physics is more complicated/uncertain



# Hydrodynamics: Numerical Methods

## Smoothed-Particle-Hydrodynamics (e.g. GADGET)

widely used in cosmological applications because of Lagrangian character

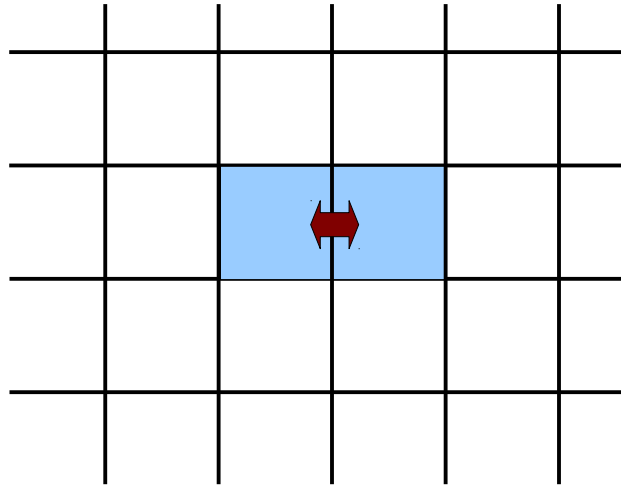


$$F_s(\mathbf{r}) = \int F(\mathbf{r}) W(\mathbf{r} - \mathbf{r}', b) d\mathbf{r}'$$

$$F_s(\mathbf{r}) \simeq \sum_j \frac{m_j}{\rho_j} F_j W(\mathbf{r} - \mathbf{r}_j, b)$$

## Finite-Volume-Methods (e.g. ENZO)

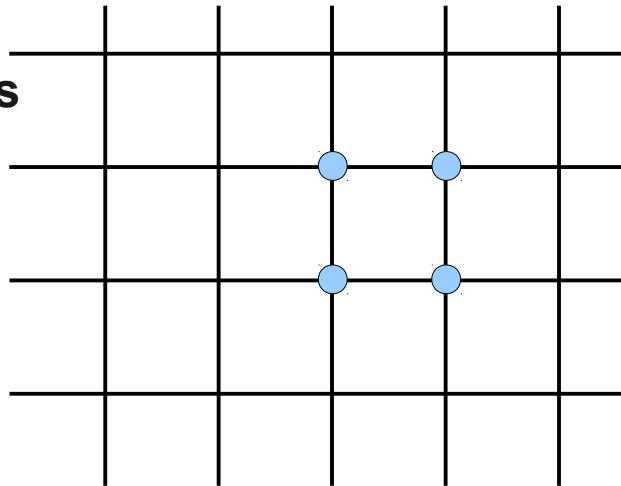
typically Eulerian approach implemented as AMR



$$\mathbf{Q}_i = \begin{pmatrix} m_i \\ \mathbf{p}_i \\ E_i \end{pmatrix} = \int_{V_i} \mathbf{U} dV$$

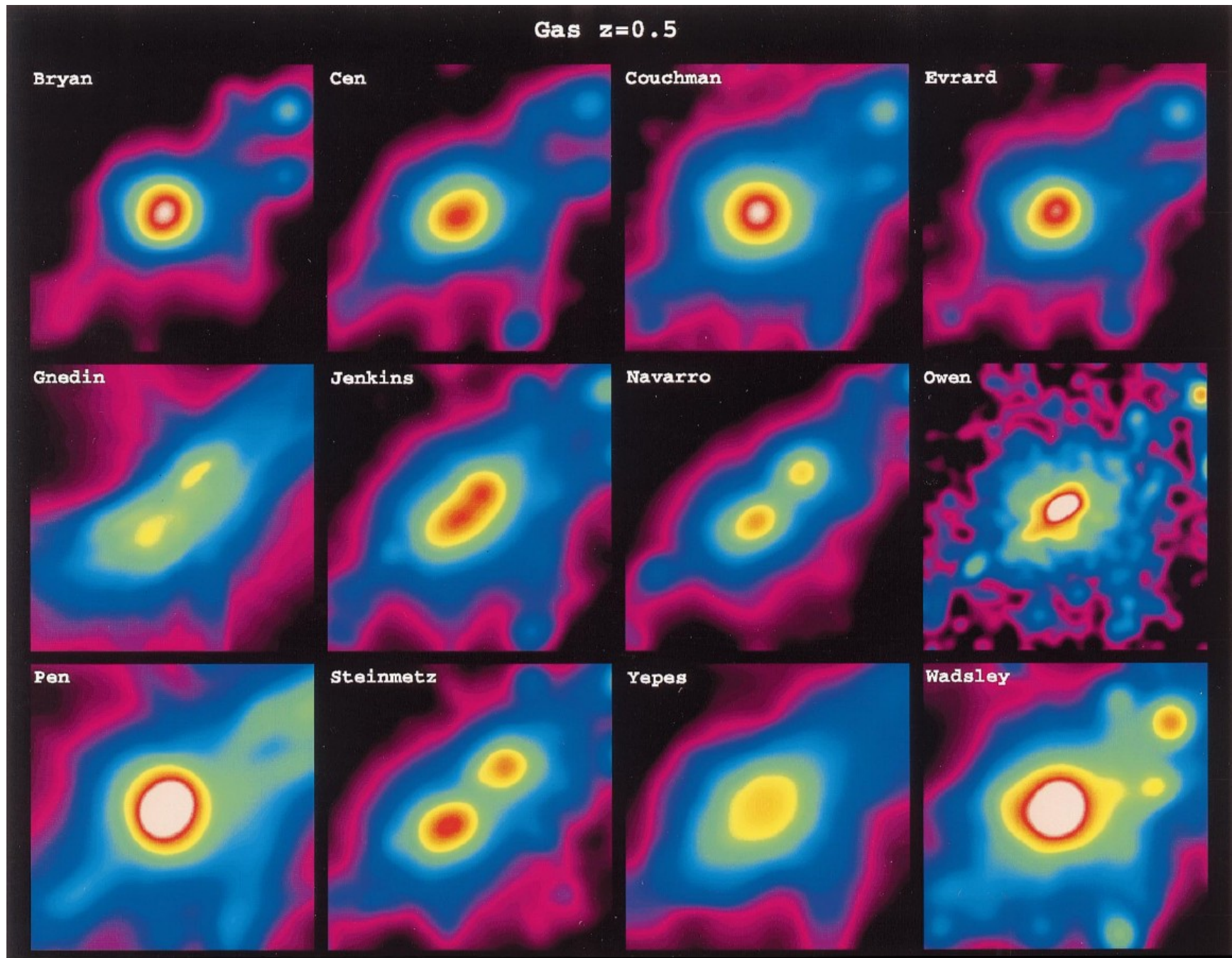
$$\frac{d\mathbf{Q}_i}{dt} = - \sum_j A_{ij} \mathbf{F}_{ij}$$

## Finite-Differencing-Methods





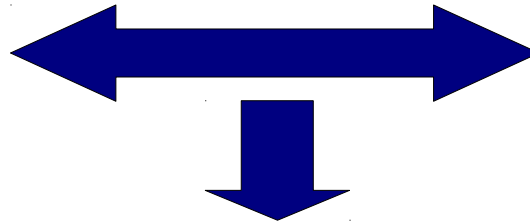
# Uncertainties



Frenk et al (1999)

# Moving Mesh Hydrodynamics

Lagrangian Methods  
(SPH)  
e.g.: Gadget



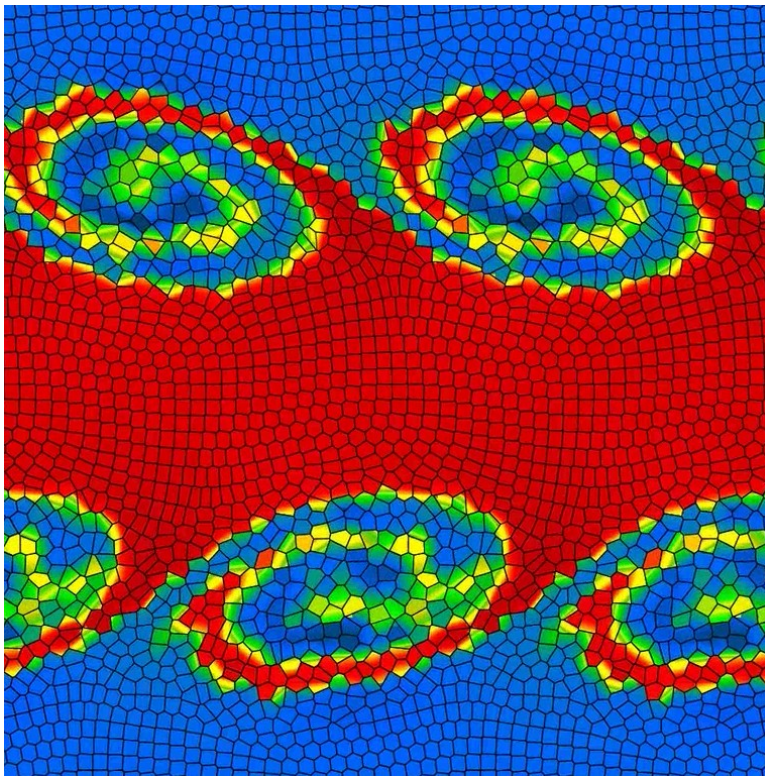
Eulerian Methods  
(AMR)  
e.g.: Enzo

Quasi-Lagrangian Hybrid Scheme: AREPO (Springel, 2010)  
TESS (Duffell & MacFadyen, 2011)

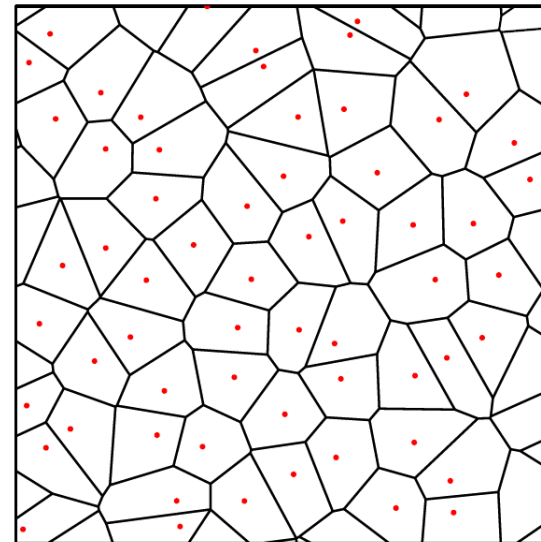
tests demonstrated that AREPO seems to work  
very well compared to other hydro schemes



**How does it perform  
on 'real' problems?**

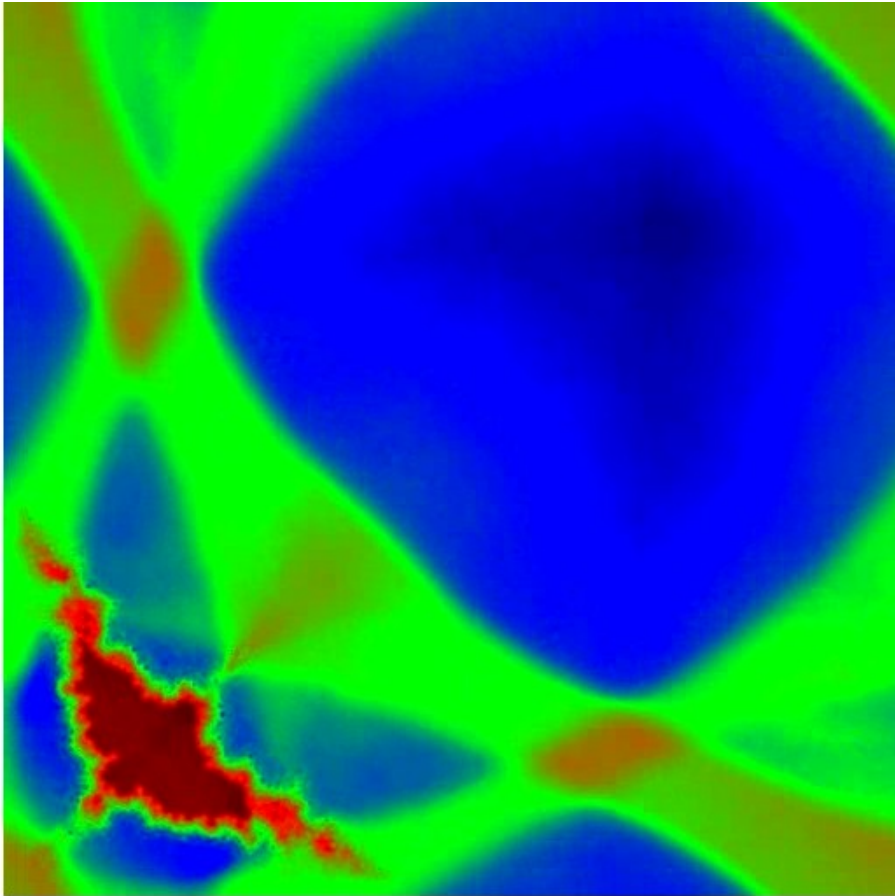


Moving Voronoi Mesh



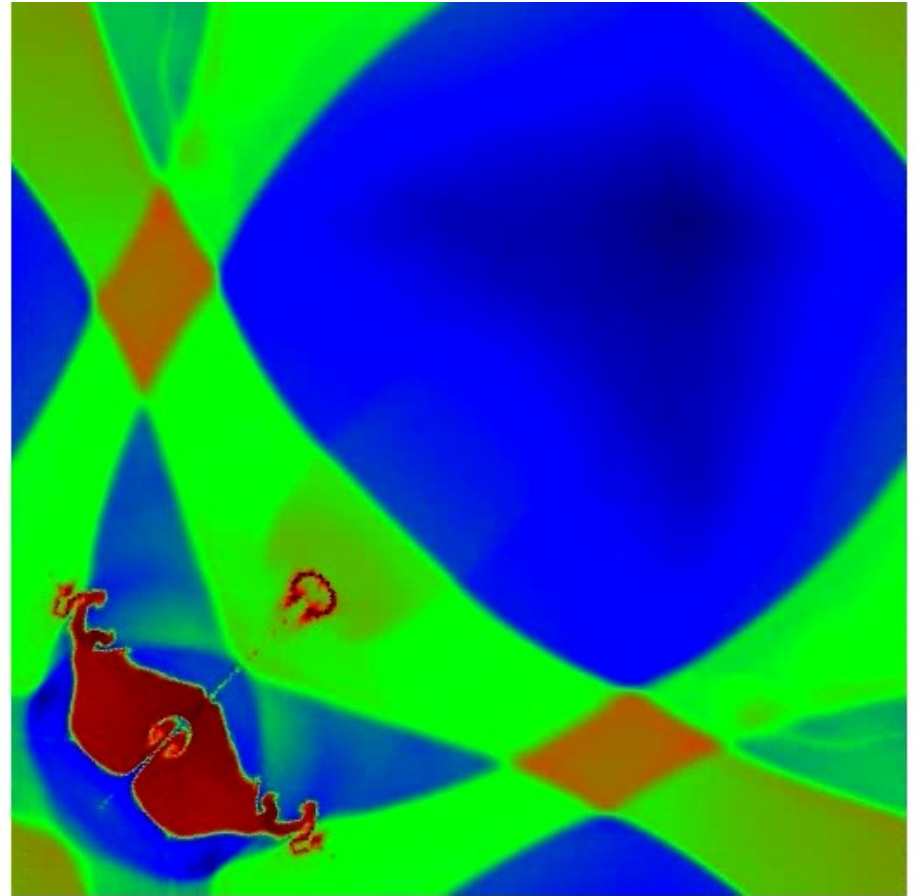


# Test Problem I



**SPH**

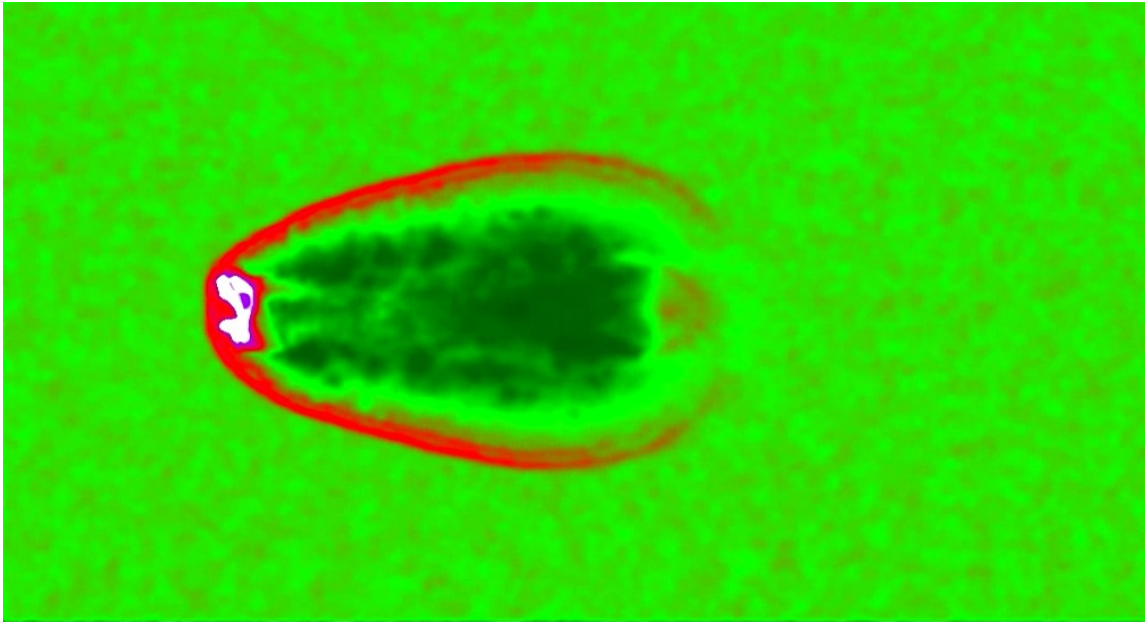
2D shocktube → interacting shocks



**MOVING MESH**

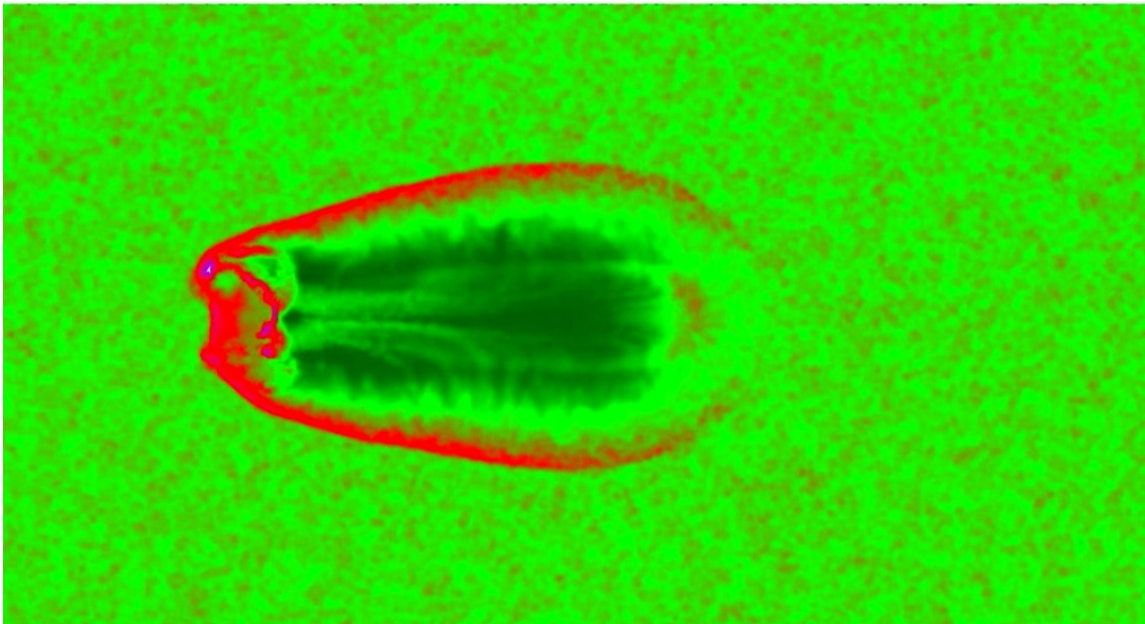
Sijacki et al (in prep)

# Test Problem II



**SPH**

Blob test



**MOVING MESH**

Sijacki et al (in prep)



# Cosmological Simulations

How does AREPO perform for cosmological problems?

What are the differences to previous SPH runs?

What are the implications for the modeling of sub-resolution physics?

## Test code and compare to SPH:

- sub-resolution physics is *identical*
- gravity solver is *identical*



Direct comparison possible

Only difference: hydro solver: **MOVING MESH vs. SPH**



**What differences are caused by new hydro scheme?**

# Simulation Setup

## Cosmology:

•  $\Omega_M=0.27$ ,  $\Omega_L=0.73$ ,  $\Omega_B=0.045$

•  $\sigma_8=0.8$ ,  $H_0=70$  km/s/Mpc

## Implemented Physics:

• Radiative Cooling: primordial mixture of H and He

• UV Background (updated Haardt&Madau)

• Star Formation/Feedback: Springel&Hernquist (2003)

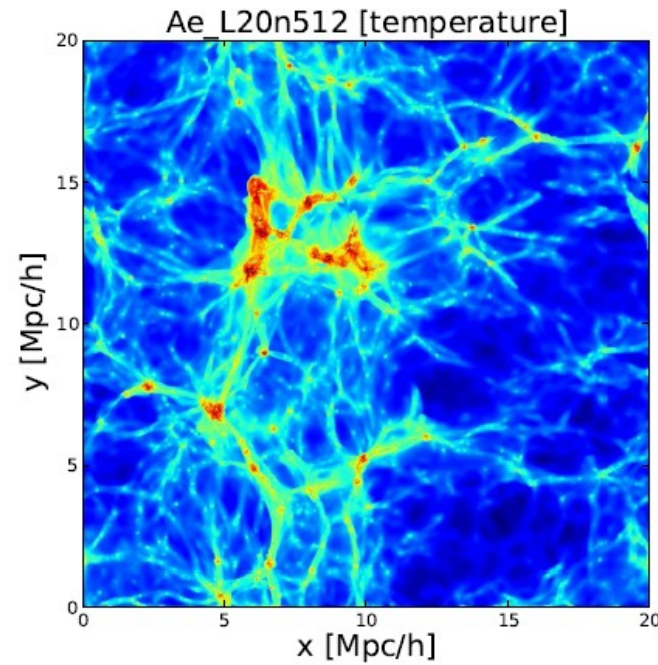
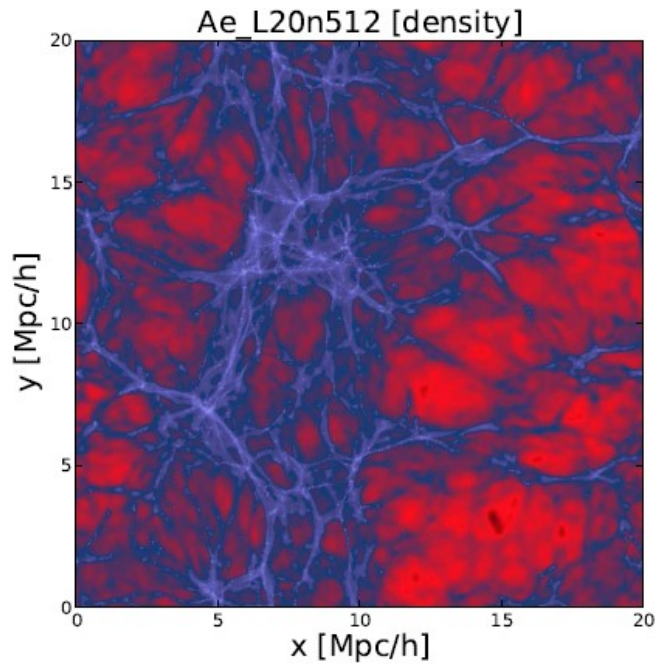
## AREPO:

- de-/refinement
- mesh regularization

MV et al (in prep)  
Keres et al (in prep)  
Sijacki et al (in prep)

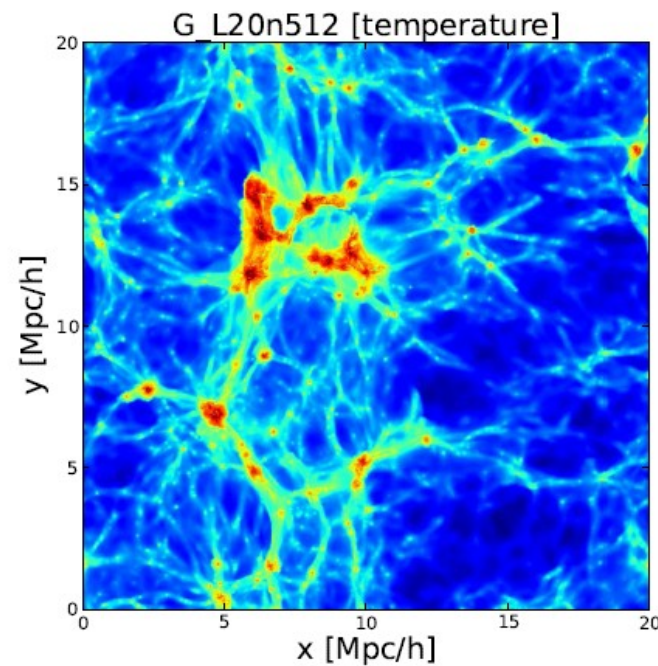
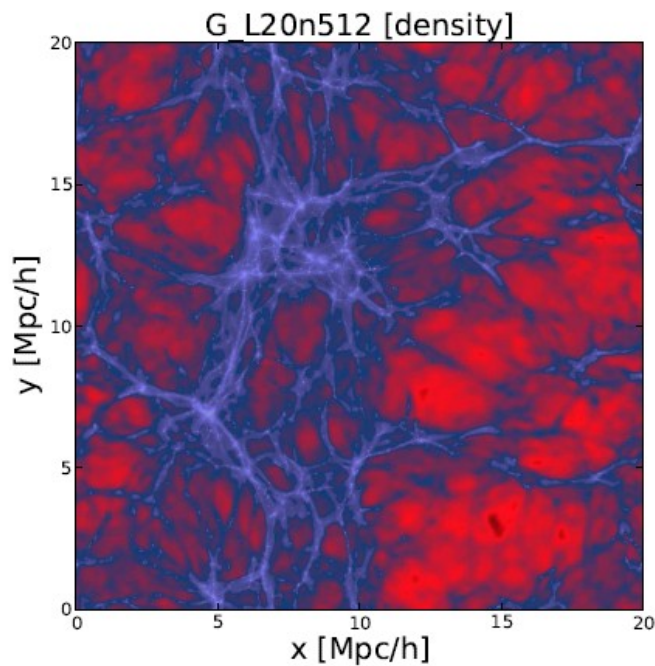
Name	Code (mode)	Boxsize[(Mpc/h) <sup>3</sup> ]	hydro elements	DM particles	$m_{DM}[M_\odot/h]$	$m_{target}/SPH[M_\odot/h]$
Ad_L20n512	AREPO (dual entropy)	$20^3$	$512^3$	$512^3$	$7.444 \times 10^5$	$3.722 \times 10^6$
Ae_L20n512	AREPO (energy)	$20^3$	$512^3$	$512^3$	$7.444 \times 10^5$	$3.722 \times 10^6$
G_L20n512	GADGET	$20^3$	$512^3$	$512^3$	$7.444 \times 10^5$	$3.722 \times 10^6$
Ad_L20n256	AREPO (dual entropy)	$20^3$	$256^3$	$256^3$	$5.955 \times 10^6$	$2.977 \times 10^7$
Ae_L20n256	AREPO (energy)	$20^3$	$256^3$	$256^3$	$5.955 \times 10^6$	$2.977 \times 10^7$
G_L20n256	GADGET	$20^3$	$256^3$	$256^3$	$5.955 \times 10^6$	$2.977 \times 10^7$
Ad_L20n128	AREPO (dual entropy)	$20^3$	$128^3$	$128^3$	$4.764 \times 10^7$	$2.382 \times 10^8$
Ae_L20n128	AREPO (energy)	$20^3$	$128^3$	$128^3$	$4.764 \times 10^7$	$2.382 \times 10^8$
G_L20n128	GADGET	$20^3$	$128^3$	$128^3$	$4.764 \times 10^7$	$2.382 \times 10^8$

# Global Density and Temperature Structure



**MOVING MESH**

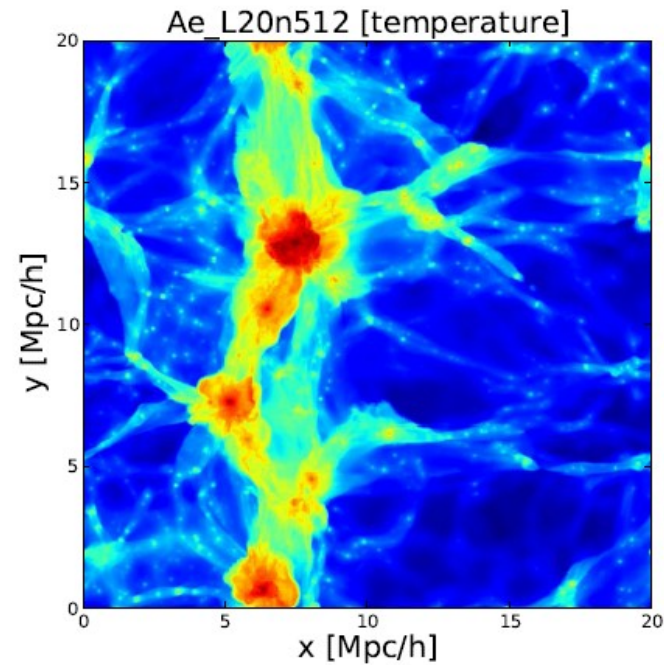
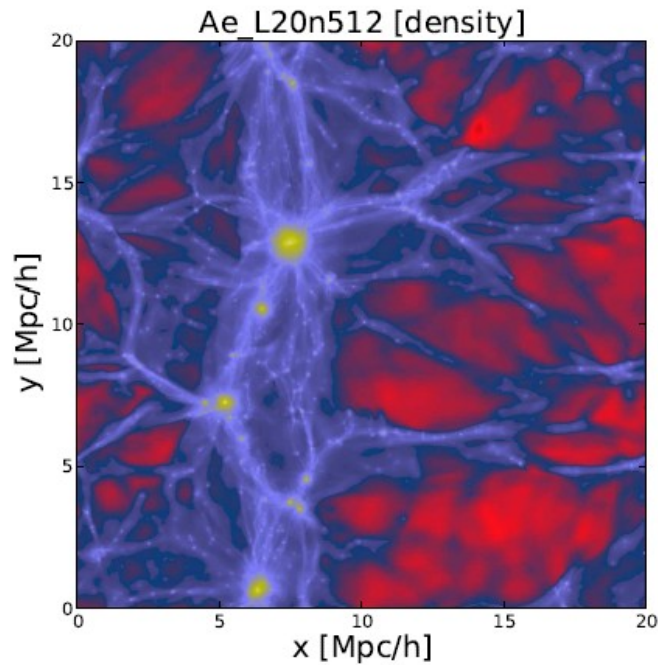
**z=2**



**SPH**

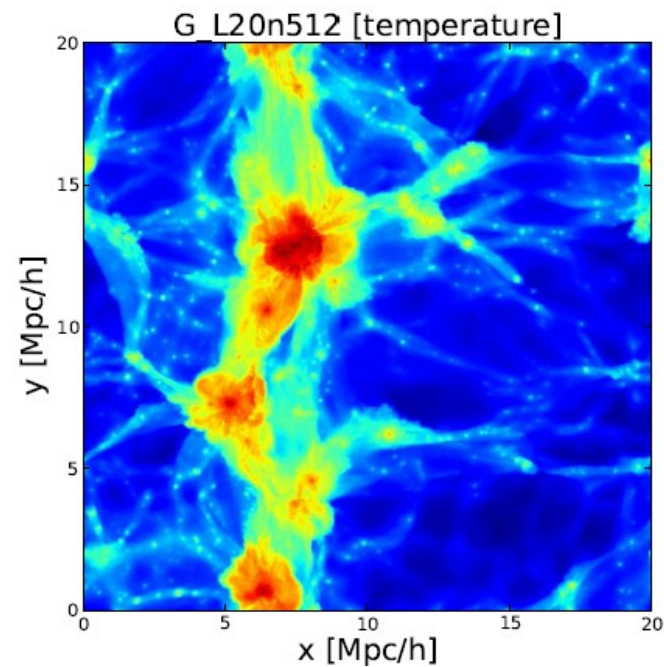
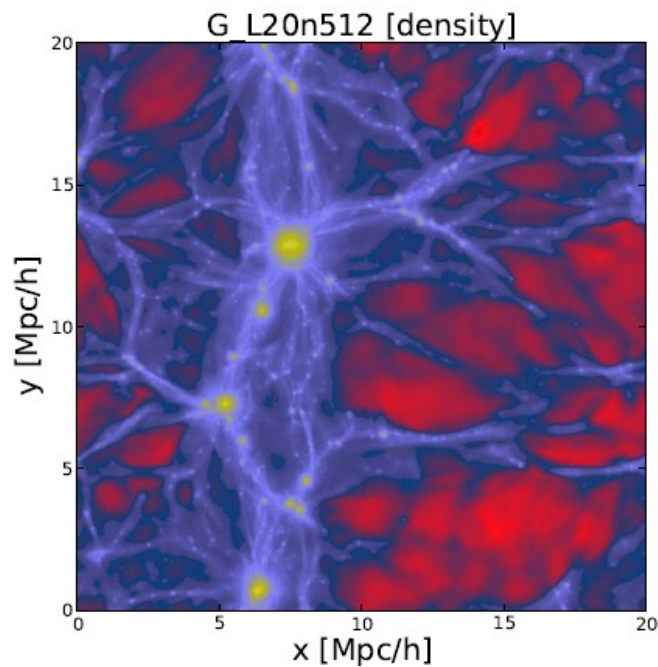


# Global Density and Temperature Structure



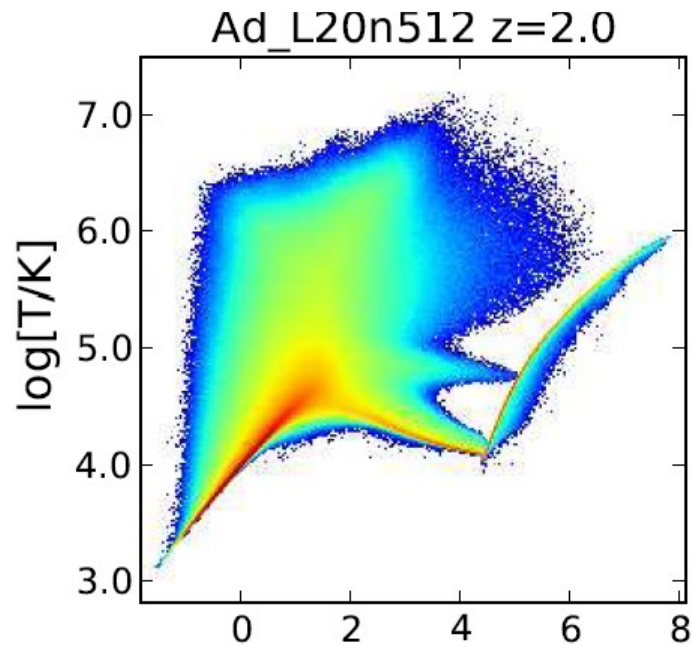
**MOVING MESH**

**z=0**

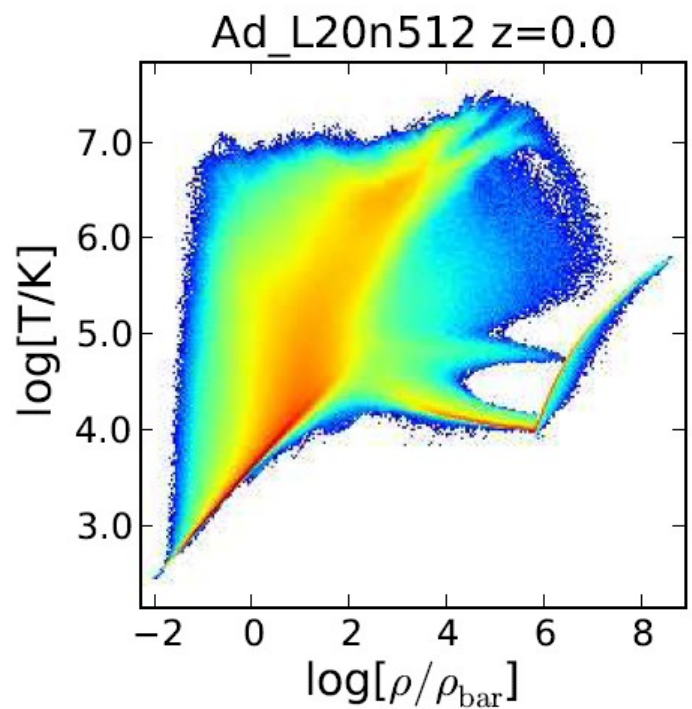
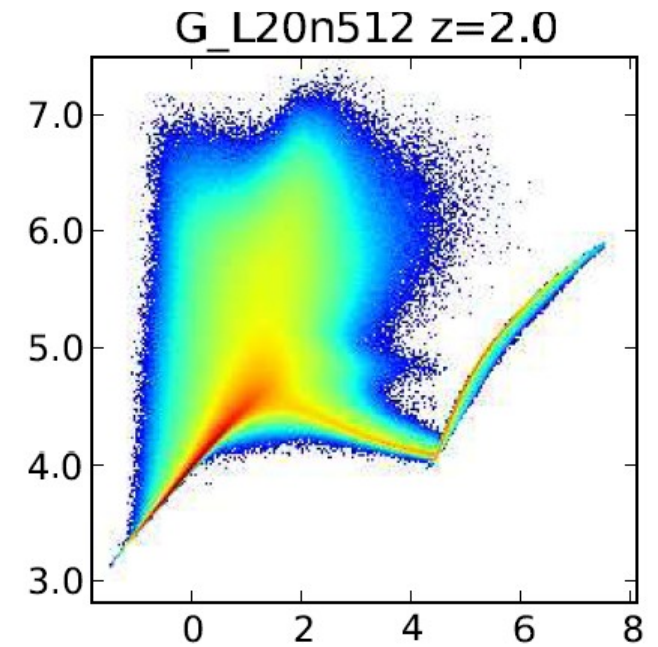
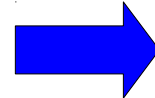


**SPH**

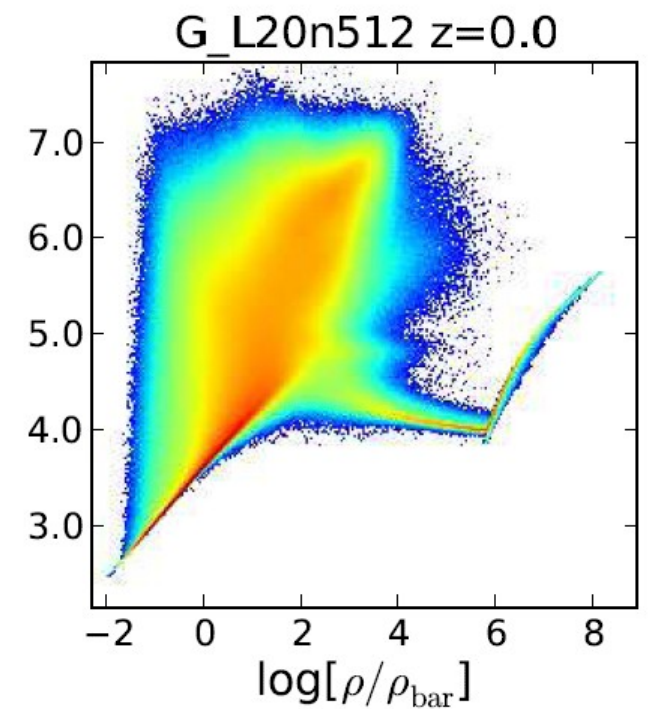
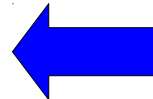
# Gas Phase Diagram



SPH

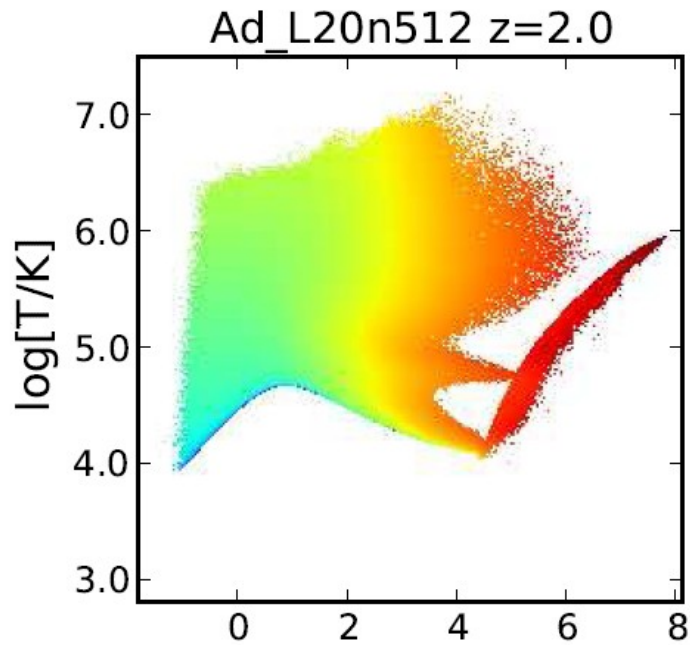


MOVING MESH

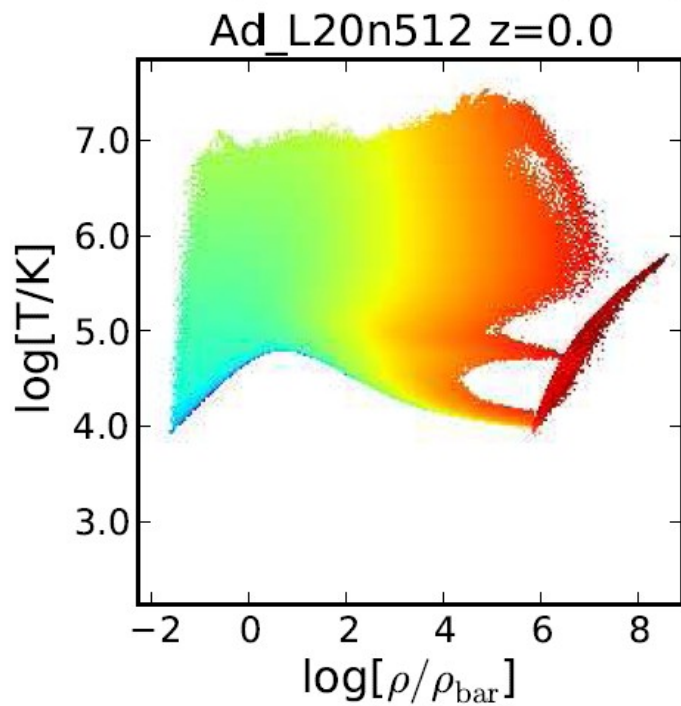
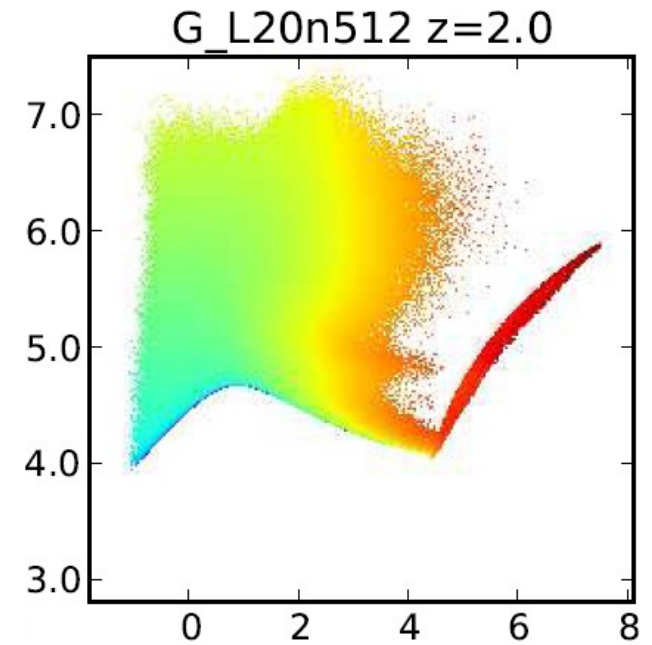




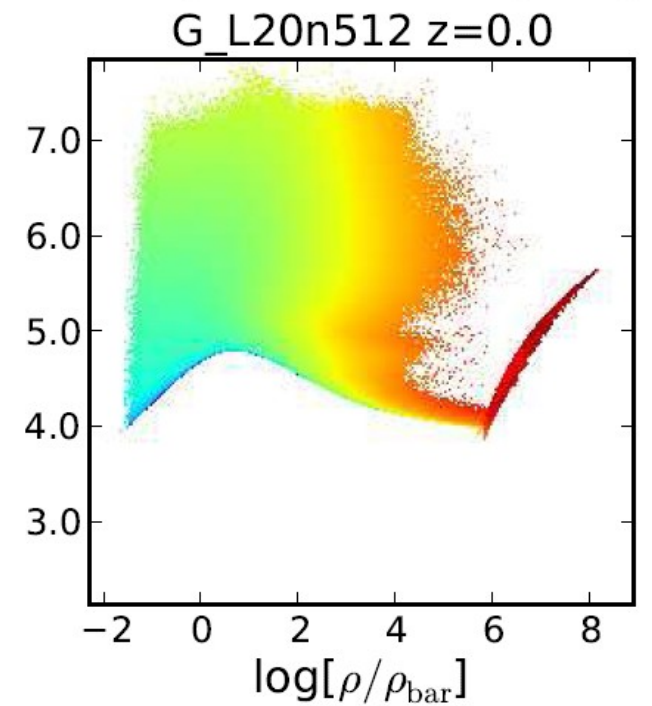
# Cooling Rates



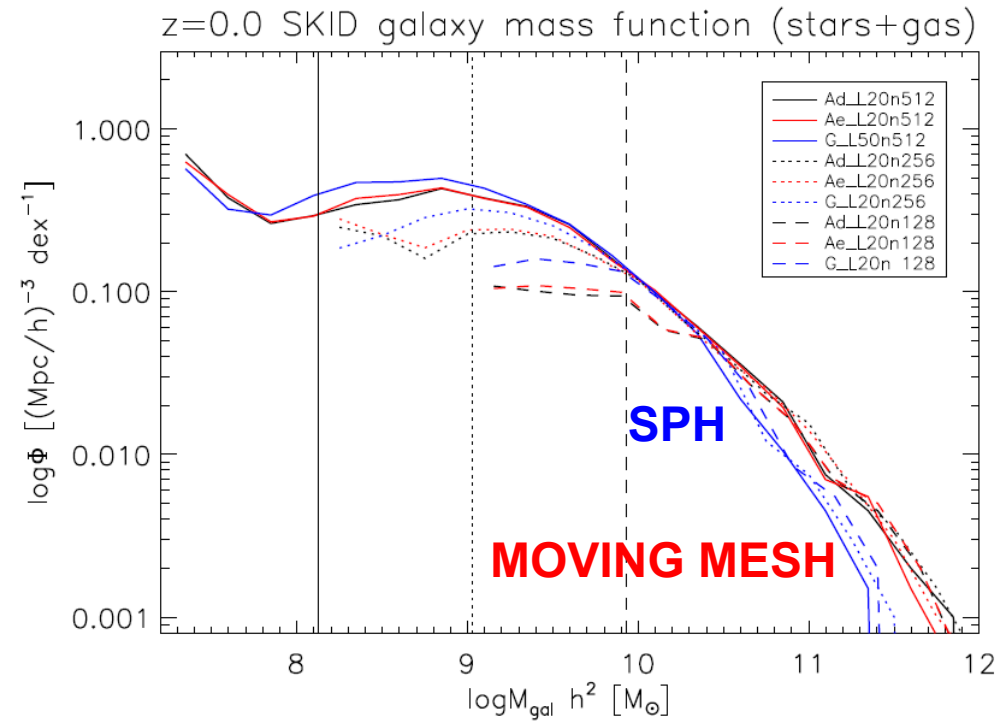
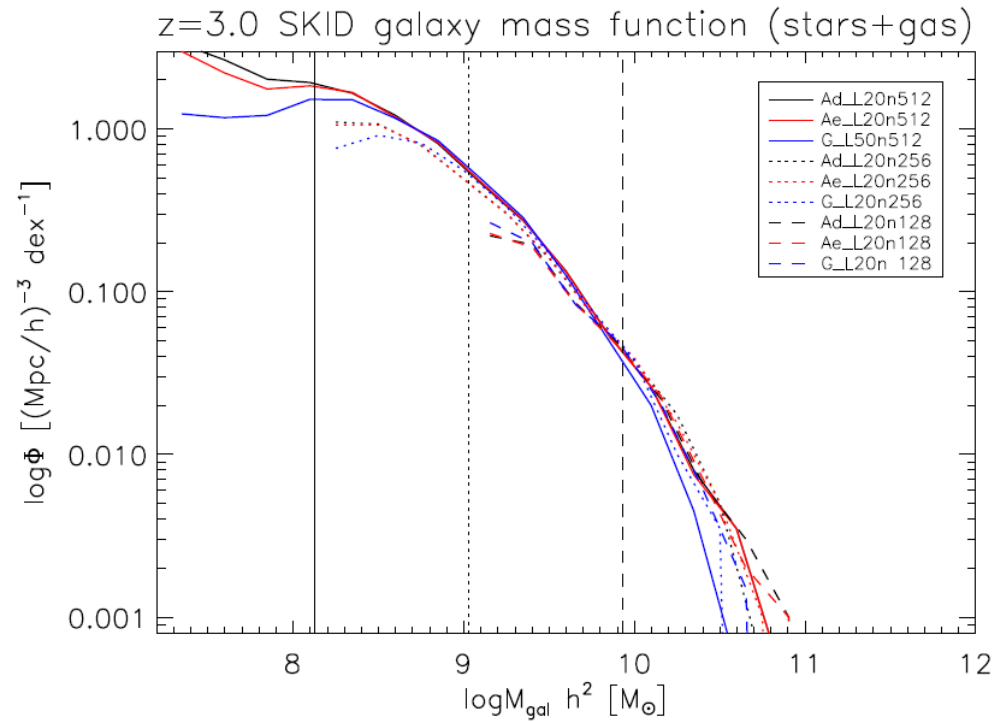
SPH 



 MOVING MESH

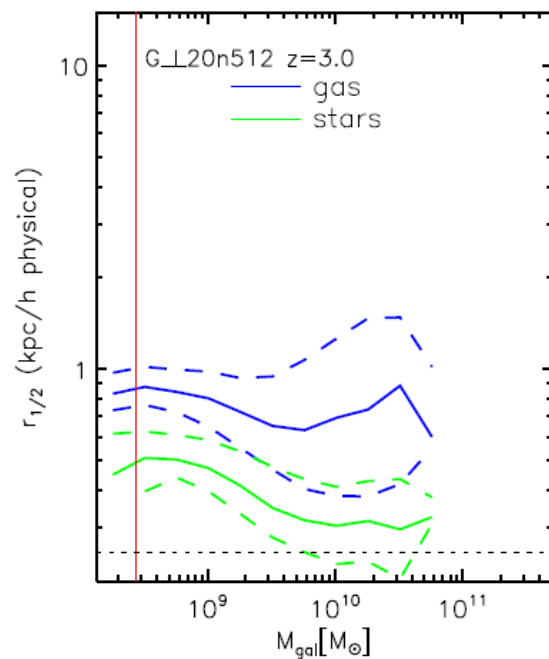


# Galaxy Mass Function

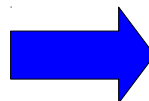


more massive galaxies in AREPO

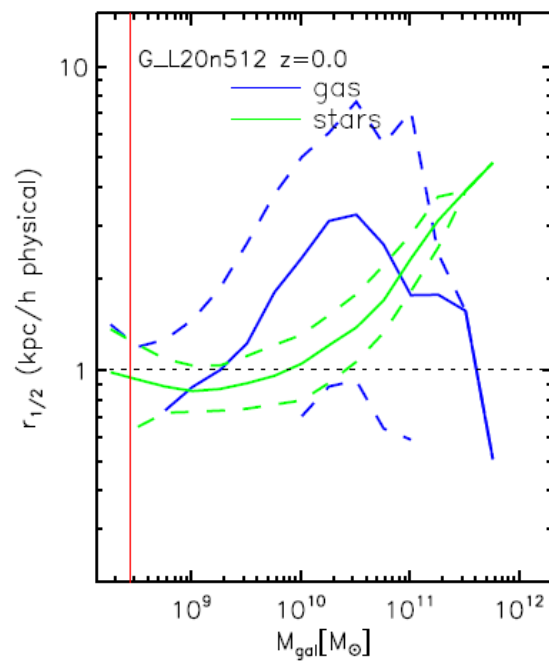
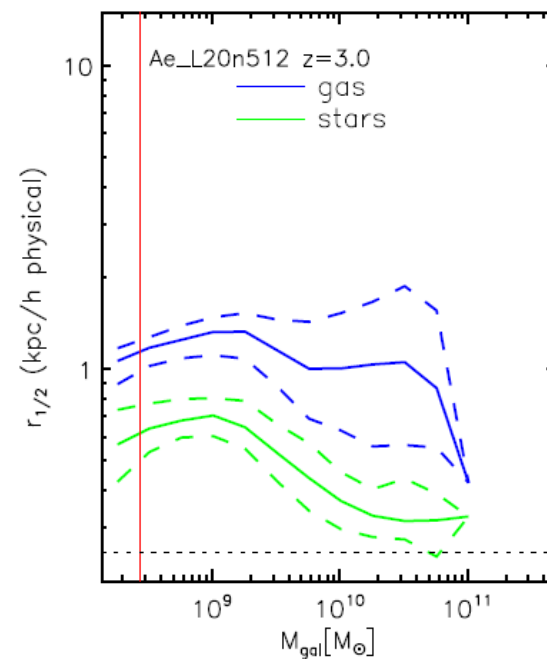
# Galaxy Sizes



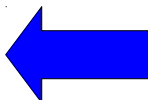
**MOVING MESH**



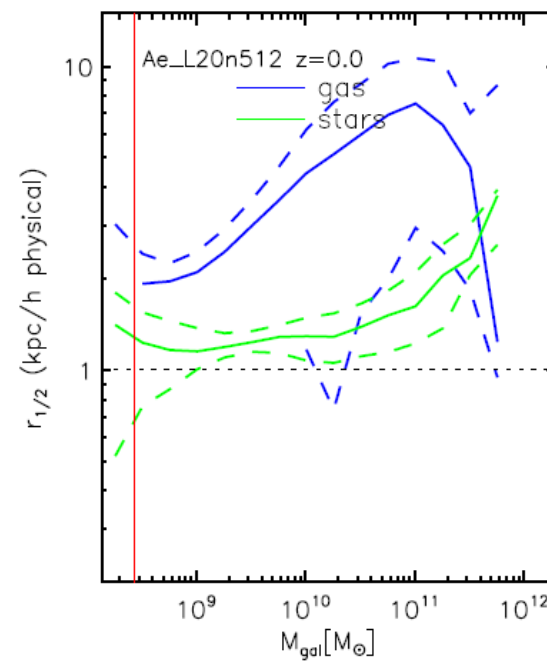
half mass radii  
typically larger  
with moving mesh



(specific angular momentum  
typically larger  
with moving mesh)

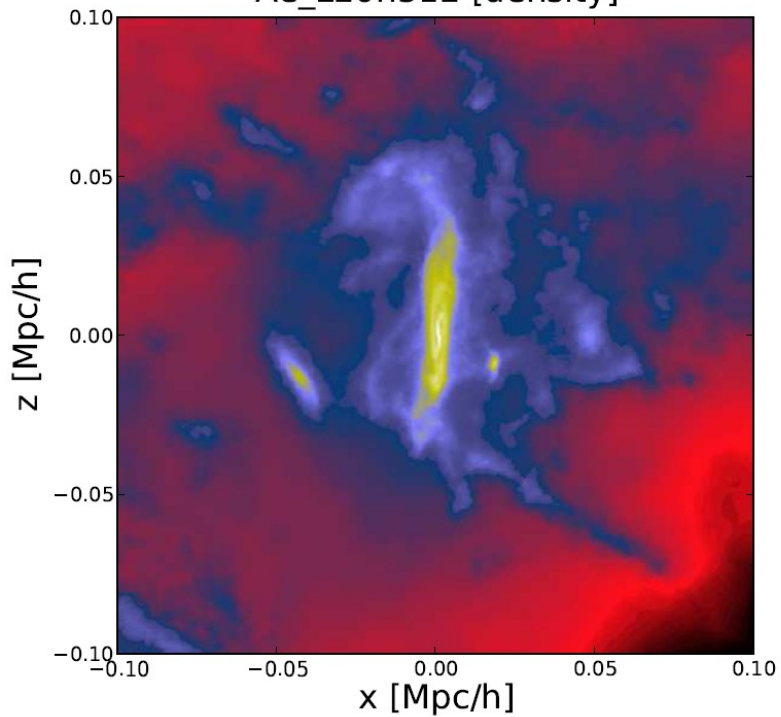


**SPH**

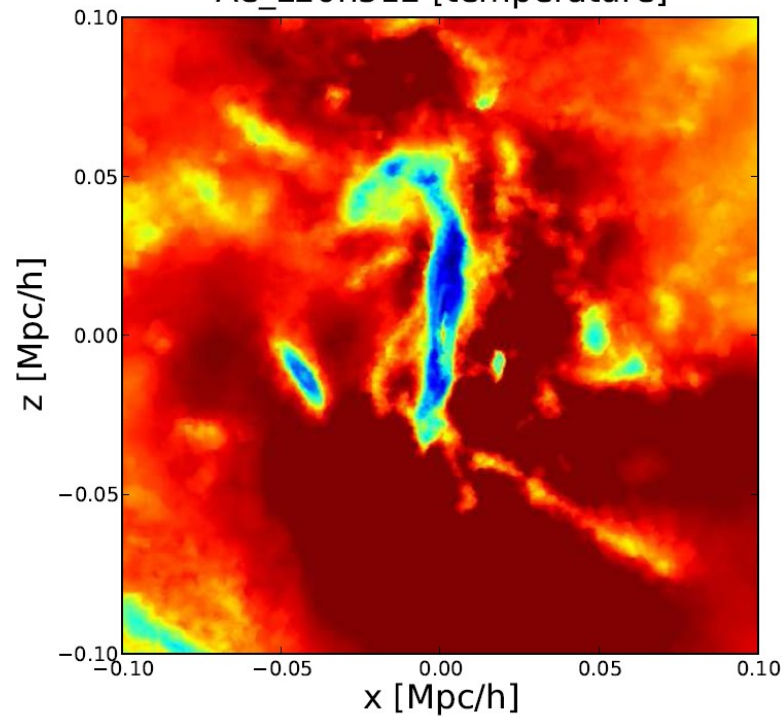


$z=2.0$

Ae\_L20n512 [density]



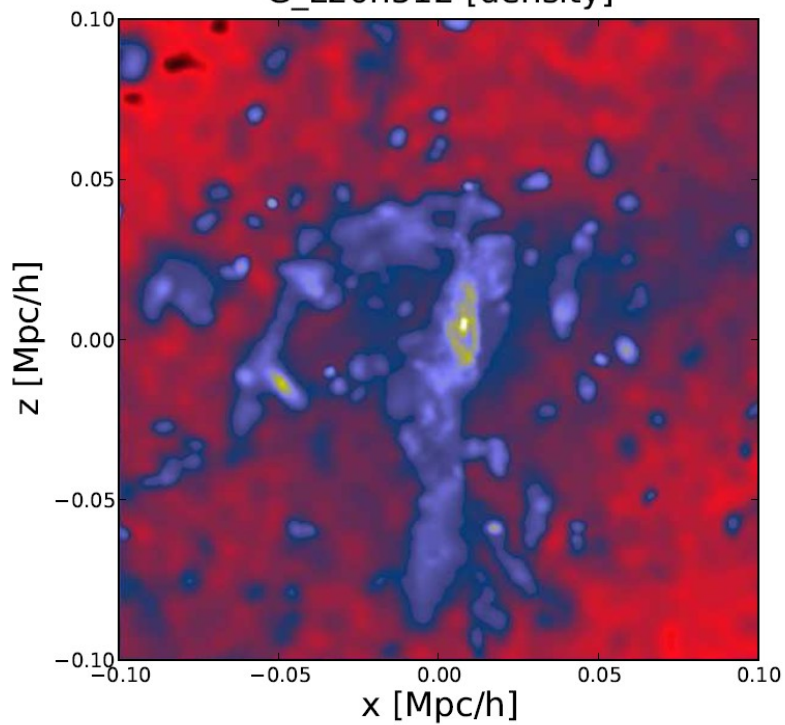
Ae\_L20n512 [temperature]



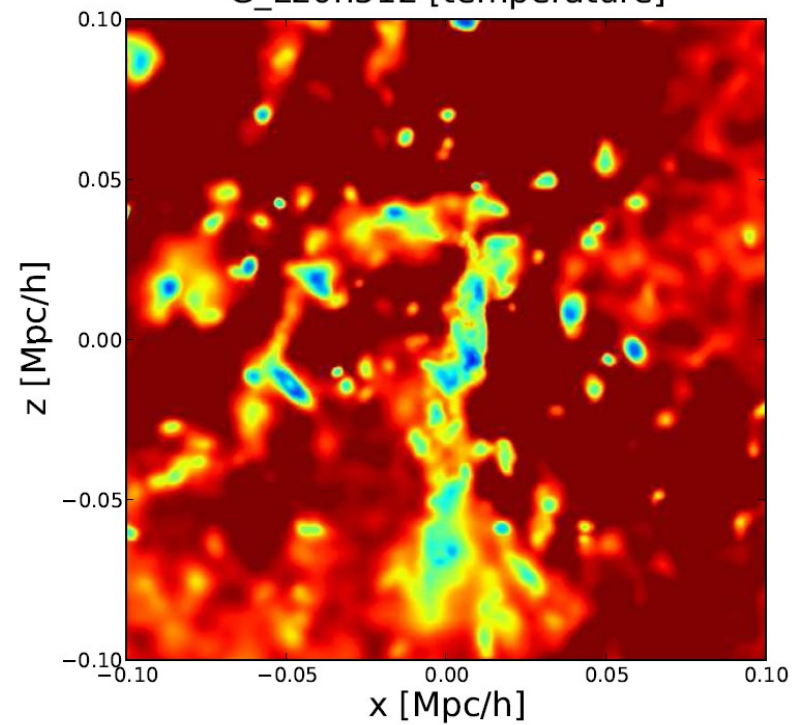
**MOVING MESH**

galaxy at  $z=2$   
edge-on

G\_L20n512 [density]



G\_L20n512 [temperature]

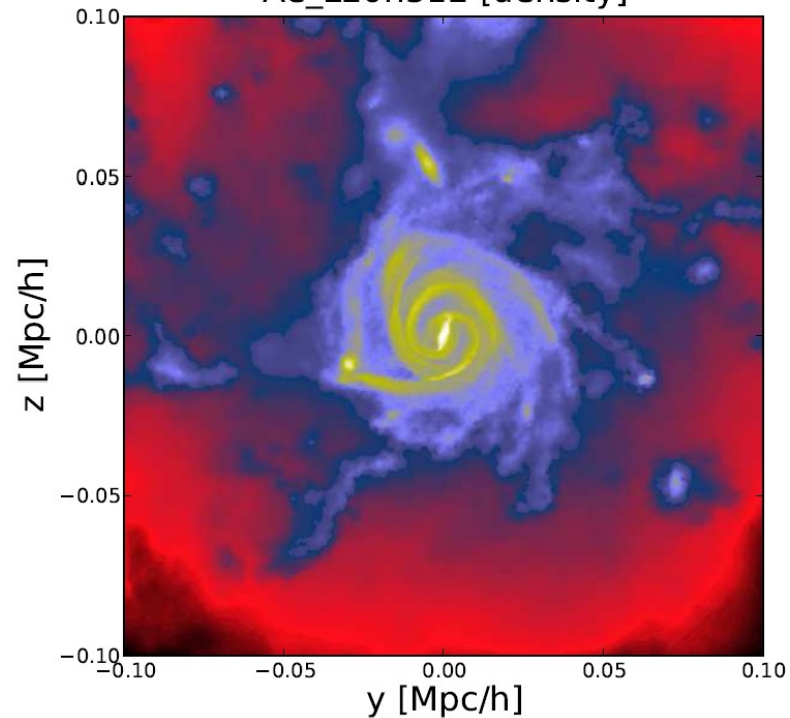


**SPH**

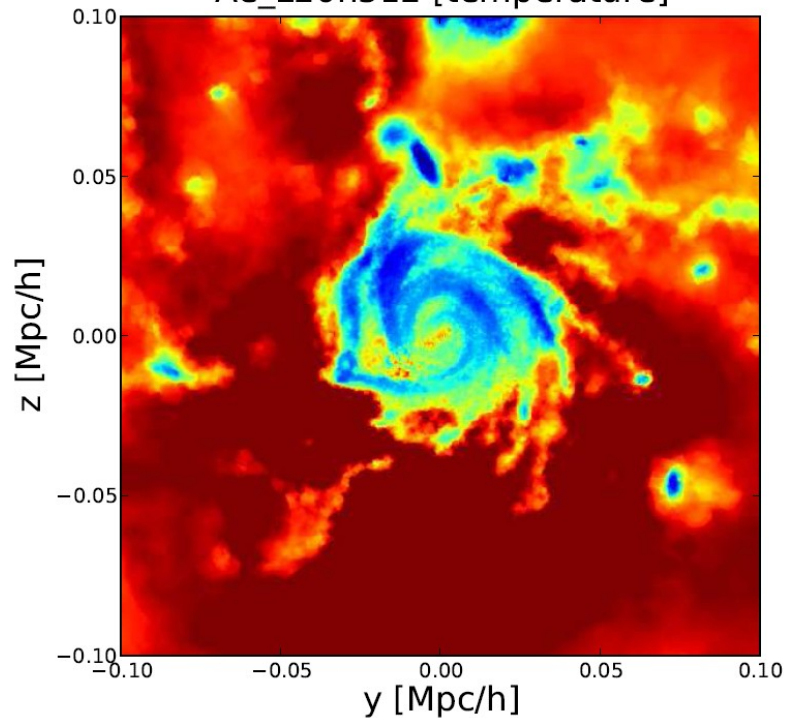


$z=2.0$

Ae\_L20n512 [density]



Ae\_L20n512 [temperature]

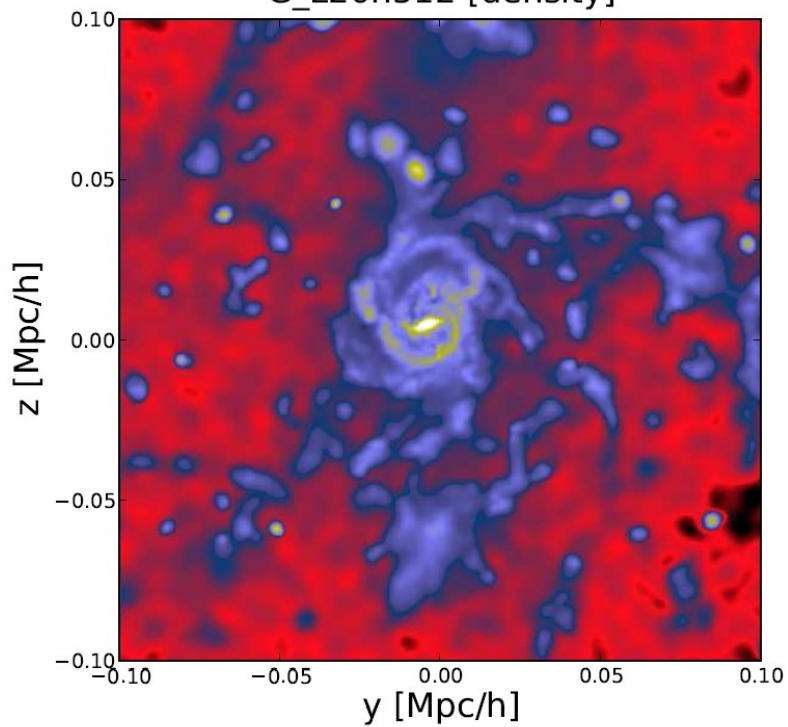


**MOVING MESH**

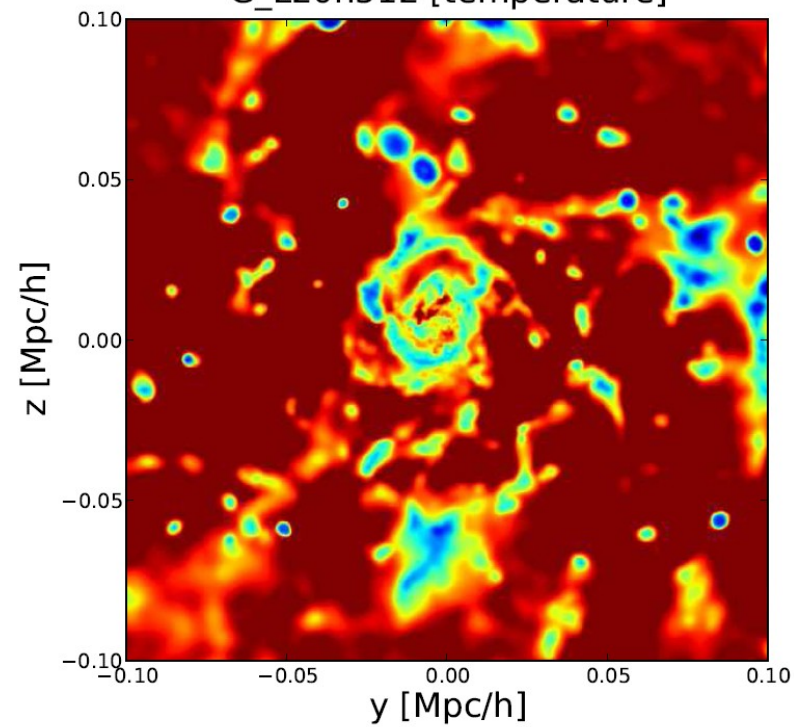
galaxy at  $z=2$   
face-on

**SPH**

G\_L20n512 [density]



G\_L20n512 [temperature]





# Performance

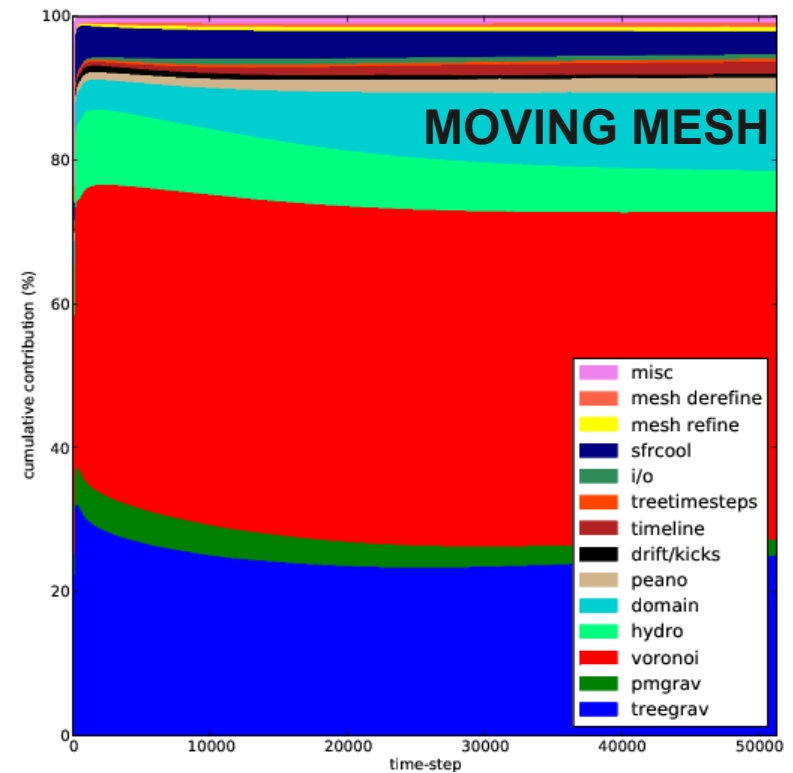
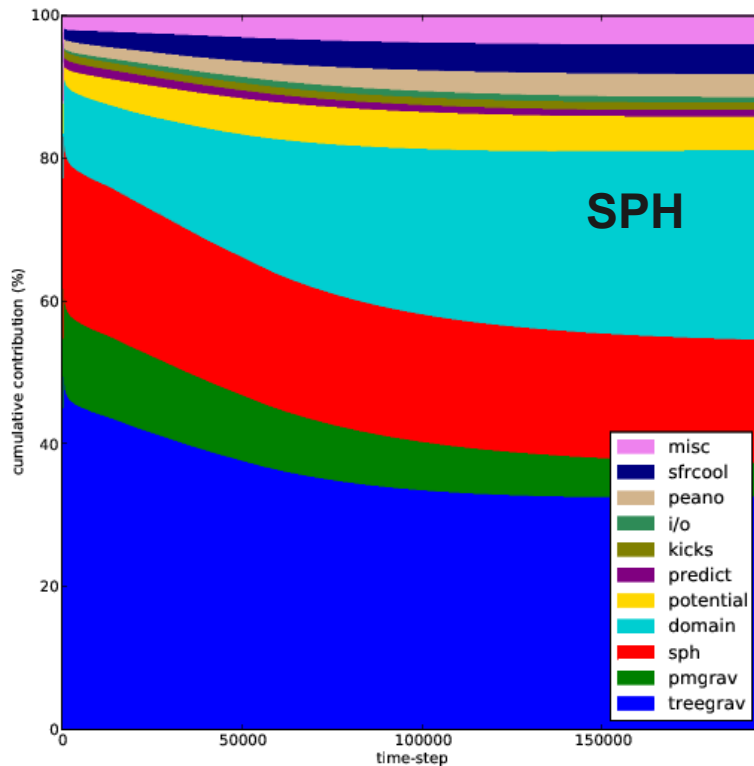
Total CPU time:

Gadget:  $2 \times 256^3 \rightarrow 64 \times 190.000\text{sec}$

Arepo:  $2 \times 256^3 \rightarrow 64 \times 240.000\text{sec}$

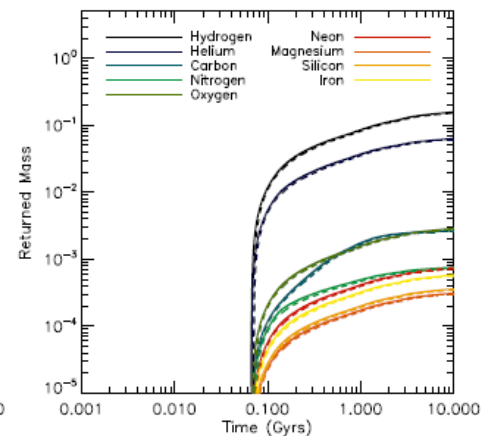
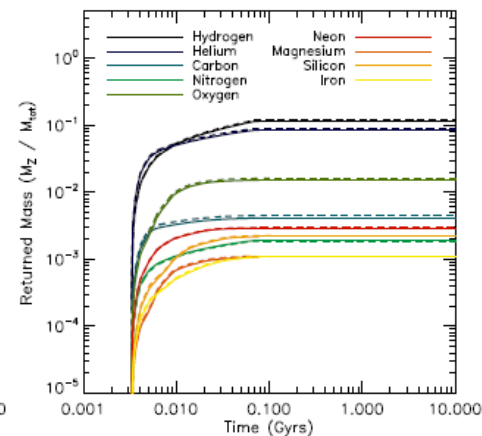
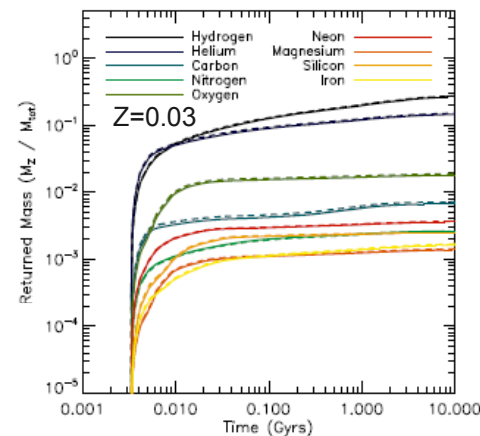
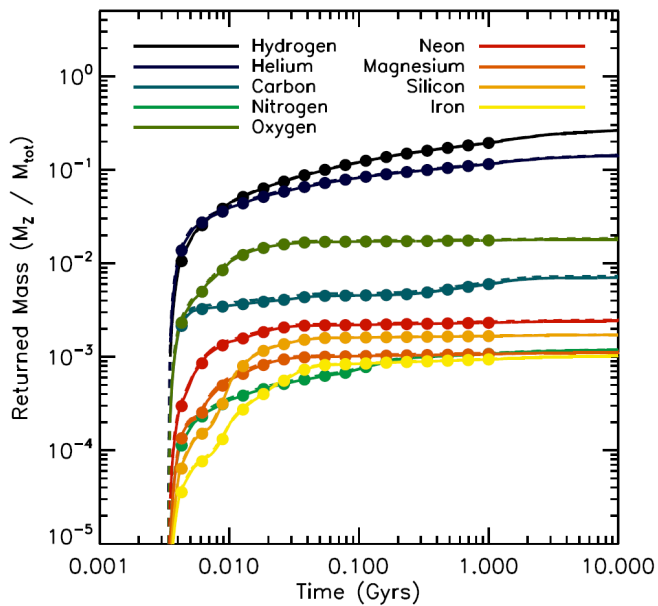
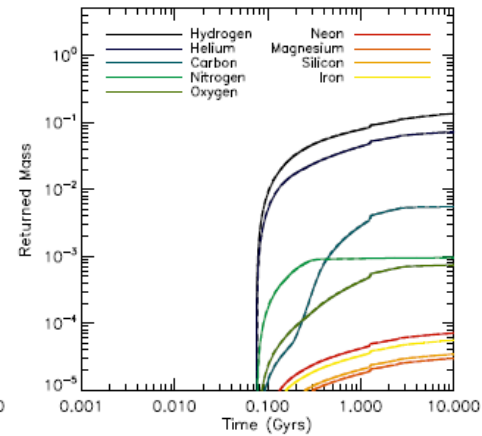
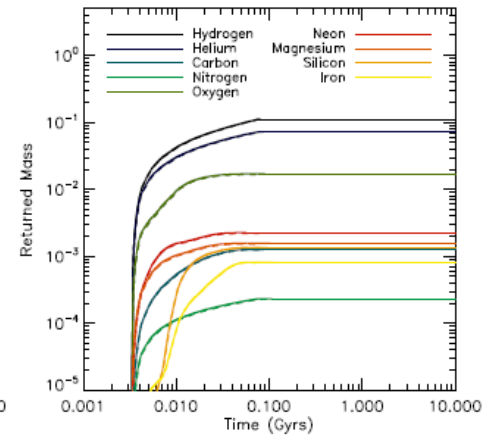
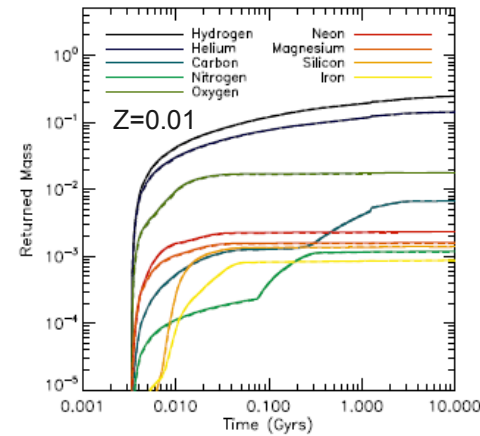
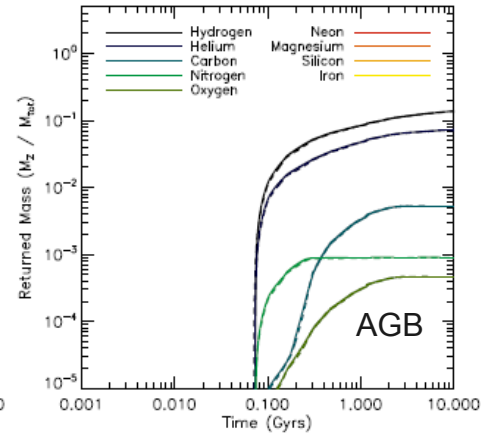
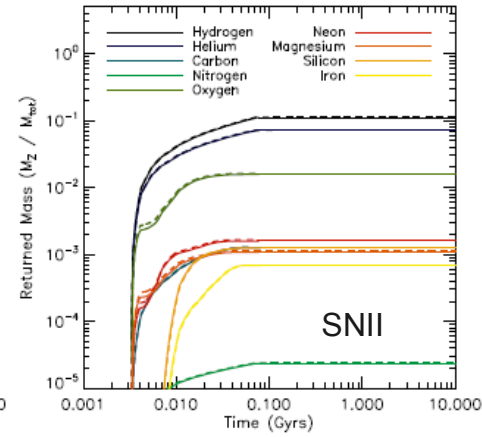
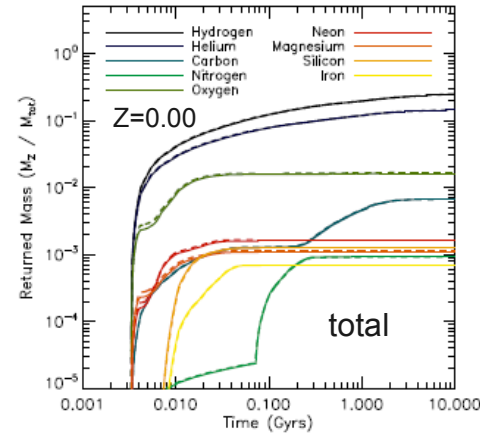


**AREPO nearly as efficient as GADGET**

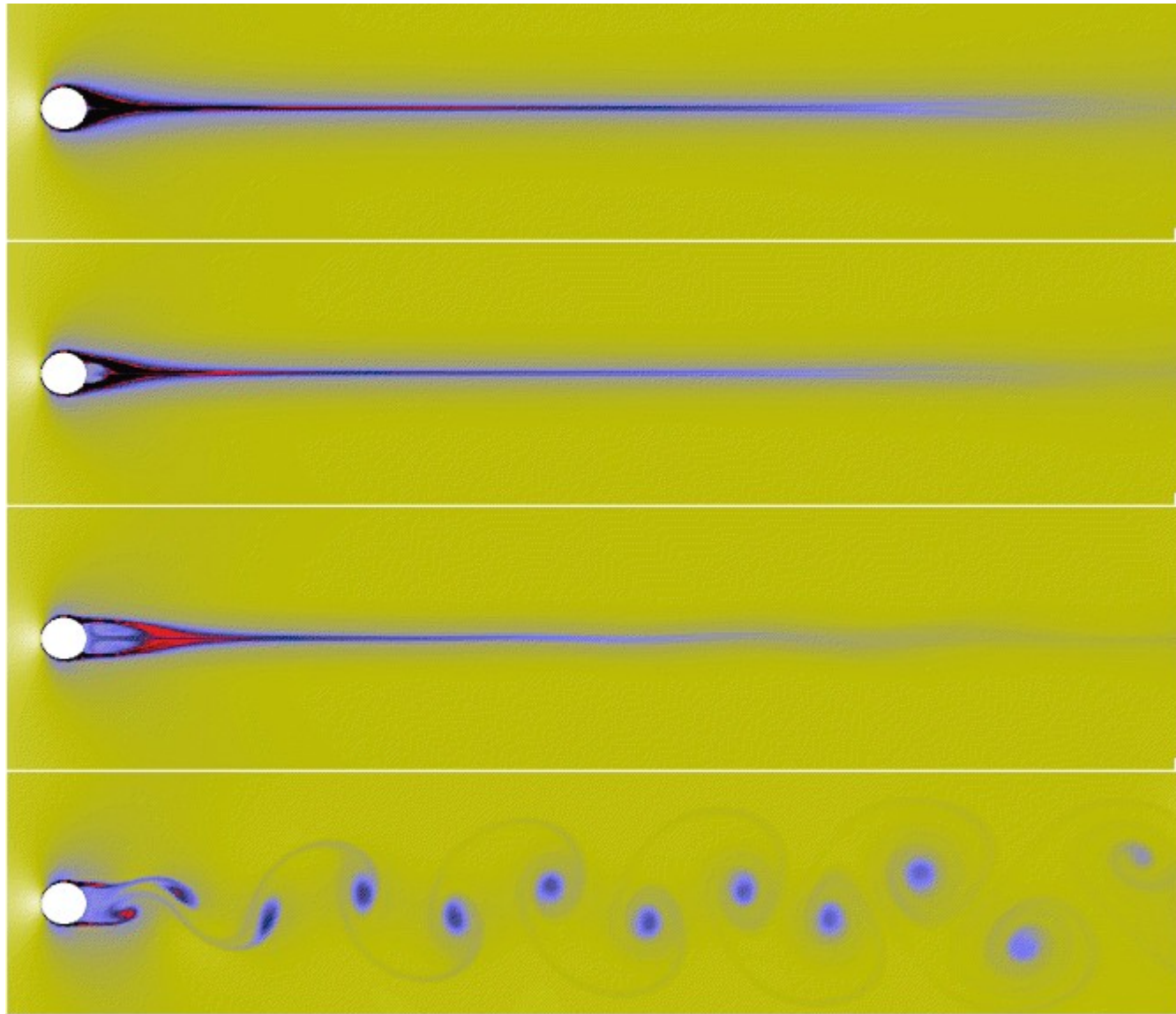


# More Physics I: Chemistry

- chemical enrichment
- stellar mass loss
- metal line cooling



# More Physics II: Navier-Stokes Terms



Re=10

Re=20

Re=40

Re=100

Munoz, Springel, Marcus, MV, Hernquist (submitted)

# Implications

- hydro-solver matters a lot (not only the modeling of sub-resolution physics)
- tuning sub-resolution models based on a wrong hydro-scheme is questionable
- AREPO offers new hydro-scheme; might lead to new insights due to its flexible setup
- performance of moving mesh schemes (work and memory) is comparable to SPH codes